Individualism, Collectivism and Economic Outcomes: A Theory and Some Evidence*

Kartik Ahuja† Mihaela van der Schaar‡
William R. Zame§

April 6, 2017

*We are grateful to John Asker, Moshe Buchinsky, Dora Costa, Stefania Innocenti, Facundo Peguillem, Alex Teytelboym, Peyton Young and seminar audiences at UCLA, Royal Holloway University, London and Oxford University for comments. Research support to Ahuja and van der Schaar was provided by the U.S. Office of Naval Research Mathematical Data Science Program; additional support to Ahuja was provided by the Guru Krupa Foundation. Support and hospitality were provided to Zame by Nuffield College, Oxford University and University College, London. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of any funding agency.

†Department of Electrical Engineering, UCLA, Los Angeles, CA 90095 USA; kartik999@gmail.com

‡Oxford-Man Institute, Oxford, OX2 6ED UK and Department of Electrical Engineering, UCLA, Los Angeles, CA 90095 USA; mihaela.vanderschaar@eng.ox.ac.uk

§Department of Economics, UCLA, Los Angeles, CA 90095 and Nuffield College, Oxford OX1 1NF, UK ; zame@econ.ucla.edu

Abstract

This paper presents a dynamic model to study the impact on economic outcomes of individualism (time spent working alone) and collectivism (complementary time spent working with others) in different societies during the Malthusian/Agricultural Era (after the Neolithic Age and before the Industrial Revolution). The model is driven by three forces: a greater degree of collectivism provides a higher safety net for low quality workers, a greater degree of individualism allows high quality workers to leave larger bequests, a larger population creates more congestion and lowers productivity. Although these forces seem simple, they interact in a very complicated way. The predictions of the model suggest that a greater degree of individualism leads to greater per capita income and to greater income inequality. Some historical evidence is consistent with these predictions.

1 Introduction

For thousands of years between the end of the Neolithic Age and the beginning of the Industrial Revolution, there was virtually no economic growth: the average growth rate of per capita income (GDP) for the world and for most societies was much less than 0.05%. (See Clark [5] for example.) But although there was no significant *growth* in per capita income *within* societies, there were significant – indeed substantial – differences in the *level* of per capita income *across* societies. These differences surely reflect the influence of geographical, technological and cultural factors; it is the influence of cultural factors that is the focus of the present paper.

This paper presents a parsimonious framework in which to explore the impact of aspects of culture on population size, per capita income (GDP) and income inequality. We construct a dynamic model to study the differential impact of collectivism (time spent working and consuming with others) versus individualism (complementary time spent working and consuming alone). The model is driven by three forces: a greater degree of collectivism provides a higher safety net for low quality workers, a greater degree of individualism allows high quality workers to leave larger bequests, and a larger population creates more congestion, which lowers productivity. These forces, although seemingly simple, interact in a very complex way and lead to a very complicated dynamical system, governed by a coupled pair of partial differential equations with moving boundary conditions. We focus on and solve for the (unique, non-degenerate) steady state of this system, which we view as a (stylized) description of the Malthusian Era – the period after the Neolithic Revolution and before the Industrial Revolution – in which there was little growth or technological change. Our model suggests that technological differences across societies are important for population size but not for per capita income or for income inequality, but that cultural differences across societies are important for all of these. In particular, our model suggests that more individualistic societies have greater per capita income and greater income inequality. These predictions are consistent with some historical evidence.

It seems to be widely agreed (see Greif [9] and Landes [13] for instance) that culture has an important influence on social outcomes and economic outcomes but there is little agreement on which aspects of culture are important for which economic outcomes, whether these aspects are different in different eras, and through what mechanisms culture operates. This paper focuses on the impact of one aspect of culture – the degree of individualism vs. collectivism – in the Malthusian Era, a period in which agriculture was the mainstay of economic activity and societies were stuck in the Malthusian trap. Many authors, including Clark [5], [6], Ashraf & Galor [1] and Galor [8] and others, have characterized this period as one of subsistence with no technological progress and little or no growth in either population or income. Our model provides a mechanism through which (some) cultural differences act, and suggests the economic consequences of these cultural differences.

We follow Hofstede [12] in viewing individualism as an aspect of culture that is associated with traits like acting independently and taking care of oneself (in particular, working and consuming alone) and collectivism as an aspect that is associated with mutual dependence amongst the members of the group (in particular, working and consuming with others). As in [12], we view individualism and collectivism as aspects of the culture of a society; as arising not from individual optimization, but from the political structure, from social norms, from environmental or other factors.[1] In our setting, this means individuals do not choose the amount of time they spend working alone or the amount of time they spend working with others. Our model is silent about how differences across societies arise or how they persist – but the data strongly suggests that such differences did exist and did persist for long periods of time.

In our model, the societal division of time/labor matters because individuals differ in productive ability (skill, etc.). When working individually, output per unit time depends on the individual's ability; when working collectively, output per unit time depends on the average ability of society – which is necessarily greater than that of low ability individuals. This implies

[1]For example, Liebbrant *et.al.* [14] show that lake based fishing areas are more individualistic and involve more isolated work by individuals, while sea based fishing areas are more collectivistic and involve more collective work by the individuals.

that a greater degree of collectivism produces two forces. On the one hand, a greater degree of collectivism provides a "social safety net" for the *current* low ability individuals. On the other hand, a greater degree of collectivism also decreases the wealth of high ability members of society and hence decreases the bequests they leave to *future* low ability individuals. Because income from production and inheritance from bequests *both* affect the path of individual wealth and hence lifespan, the degree of collectivism (and the complementary degree of individualism) create *opposing forces*; the balance of these forces (and others) plays out in a complicated and subtle way.

Clark [5], [6], Ashraf & Galor [1] and Galor [8] have offered mathematical models of the Malthusian period, but these models do not offer an explanation of how or why cross-cultural differences – in particular, differences in the degree of individualism and collectivism – might have influenced outcomes in this period. This is precisely the explanation our mathematical model is intended to provide. Gorodnichenko & Roland [10], [11] offer an analysis of the impact of individualism vs. collectivism in the era *after* the Industrial Revolution. They argue that individualism rewards status and hence promotes innovation which in turn promotes growth. However it does not seem that this explanation can explain the impact of individualism vs. collectivism in the Malthusian Era – in which there was no growth. Benabou, Ticchi & Vindigni [2], [3] argue that a different aspect of culture – religiosity – has important implications for economic outcomes in contemporary societies. Although our work is explicitly intended to address the Malthusian Era, the same approach might also have something to say about the Industrial Revolution and the transition to modern societies, as in Voigtlander and Voth [18, 19] for instance.

As noted above, the predictions of our model are consistent with some historical evidence. In particular, we offer some (unfortunately, not very good) data on per capita income and the income distribution for a range of societies and regress against Hofstede's [12] measures of individualism to argue that our model, although certainly highly stylized, is at least consistent with this historical evidence: the R^2 for per capita income is of 0.25 and the R^2 for the Gini coefficient (the obvious measure of income distribution) is

0.63. We discuss our prediction about the irrelevance of technology in the context of the data and model offered by Ashraf & Galor [1].

The remainder of the paper is organized in the following way. In Section 2 we begin with an expanded but still informal verbal description, and then lay out the formal mathematical model. Section 3 defines a steady state of the model, lays out some technical conditions and then shows that, under those conditions, there exists a unique non-degenerate steady state. Section 4 collects the main theoretical predictions of our model. Section 5 presents and discusses some historical evidence. Section 6 concludes. All proofs are collected in Appendix A; an extension of the basic model is sketched in Appendix B.

2 Model

The features of the model that we develop here are intended to represent (some aspects of) steady-state outcomes of societies in the Malthusian Era, in which (changing) technology does not play an important role.

2.1 Informal Description

Before giving a formal mathematical description of the model, we begin with an informal verbal description that expands on what we have already said in the Introduction. We consider a world populated by a continuum of individuals of two types either Low quality or High quality.[2] Time is continuous and the horizon is infinite. The lifecycle of an individual is:

- individuals are born and come into an inheritance;
- during their lifetimes, individuals consume and produce;
- individuals die and leave a bequest for succeeding individuals.

[2] Allowing for more quality levels would complicate the analysis without altering the qualitative conclusions.

While they are alive and producing, each individual spends a fraction of its time working alone and consuming the output of its individual production, and the complementary fraction of its time working with others and sharing (equally) in the joint production. We interpret these fractions as (proxies for) the *degree of individualism* and the *degree of collectivism* of the society. We view these fractions as norms which are the same across all individuals in the society, rather than as individual choices. (We are agnostic about the origins of these norms; one possibility is that they are imposed by a governmental structure, another is that they represent a social norm, a third is that they arise from the necessities of the environment – but there are many other possibilities.) When individuals work alone, their output depends on their own quality; when individuals work with others, their output depends on the average quality of society. In both modes, output is subject to congestion: productivity is less when the total population is greater. (This congestion is an essential part of Clark's argument for why societies remain in the Malthusian trap and plays an important role in our model as well.) During their lifetimes, individuals produce and consume. Some individuals produce less than they consume and eventually consume their entire inheritance; at that point their wealth is zero and they die in poverty. Individuals who do not die in poverty eventually die of natural causes. Individuals who die with positive wealth leave that wealth as a bequest to the new-born.

2.2 Formal Description

We now turn to the formal mathematical description. We consider a continuous-time model with a continuum of individuals. Some individuals are of High quality and some are of Low quality; it is convenient to index quality by $Q = 0, 1$ (Low, High).[3] The state of society at each moment of time is described by the *population distributions* $\mathcal{P}_0, \mathcal{P}_1$; $\mathcal{P}_Q(x, t)$ is the population of individuals of quality Q who have wealth less than or equal to x at time t.

[3]The individuals in our model are productive adults, so we view their quality as fixed and not changing over their lifetimes.

The *population of individuals of quality Q at time t* is

$$P_Q(t) = \lim_{x\to\infty} \mathcal{P}_Q(x,t)$$

Thus the *total population at time t* is

$$P(t) = P_0(t) + P_1(t)$$

and the *average quality at time t* is

$$\bar{Q}(t) = \frac{[0 \cdot P_0(t) + 1 \cdot P_1(t)]}{P(t)} = \frac{P_1(t)}{P(t)}$$

Individuals are born at the constant rate λ_b and die natural deaths at the constant rate λ_d. (Clark [5] argues that the fertility rate is an increasing function of the wealth of society and that the death rate is a decreasing function of the wealth of society. Those features could be incorporated into our model without changing the qualitative conclusions, although at the expense of mathematical complication. However, in the steady-state, which is our focus, birth and death rates would – by definition – be constant in any case.) Half of all newborns are of High quality and half are of Low quality. (The assumption that the proportions of new-borns of High and Low quality are constant is made only for simplicity: none of the qualitative results would change if we assumed that quality is partly inheritable, so that the proportions of High and Low quality newborns depend on the current population. Again, in the steady state, the proportions would be constant in any case. The assumption of equal proportions is made only to simplify the algebra.) As we discuss below, some individuals also die in poverty.

While they are alive, individuals produce and consume. We assume that each individual spends a fraction z of its time working alone and the remaining fraction $1-z$ working with others. As noted, we identify z with the *degree of individualism* of the society and $1-z$ as the *degree of collectivism.* When an individual works alone its production depends on its own quality and is consumed entirely by the individual; when it works with others its production depends on the average quality of society (at the given moment of time) and is shared; in both modes, productivity is subject to congestion and so diminishes with increasing population. For simplicity, we assume production is linear in quality so the production of an individual of quality $Q = 0, 1$ at a given time t when population is $P(t)$ and average quality is $\bar{Q}(t)$ is $[\pi_Q - cP(t)]$ when working alone and $[\gamma\bar{Q}(t) - cP(t)]$ when working with others, where π_0, π_1 are the parameters of individual productivity and γ is the parameter of group productivity. Hence the *overall productivity* of an individual of quality $Q = 0, 1$ is

$$F_Q(t) = z[\pi_Q - cP(t)] + (1 - z)[\gamma\bar{Q}(t) - cP(t)] \tag{1}$$

We emphasize that Q is the innate and fixed quality of the (adult) individual and that $z, 1-z$ are characteristics of the society, and not individual choices.

For algebraic convenience, we normalize so that productivities are $\pi_0 = 0, \pi_1 = 1$. We also assume that each individual consumes at the constant subsistence rate $1/2$; thus, when working alone, low quality individuals working alone produce less than they consume and high quality individuals produce more. (In Appendix B, we sketch a more general model in which the productivities and the subsistence level are parameters and individuals who produce in excess of the subsistence level consume a fraction of the excess. The essential assumption is only that, when working alone, low quality individuals produce less than they consume and high quality individuals consume more. Details of the analysis of the more general model are available from the authors on request.) Hence the rate of production net of consumption for an individual with quality Q is

$$\begin{aligned} F_Q(t) - 1/2 &= z[Q - cP(t)] + (1 - z)[\gamma\bar{Q}(t) - cP(t)] - 1/2 \\ &= zQ + (1 - z)\gamma\bar{Q}(t) - cP(t) - 1/2 \end{aligned} \tag{2}$$

The role of the parameter group productivity parameter γ will be discussed in greater detail below and in the Conclusion.

Individuals who die at time t leave a fraction $\eta \in (0, 1)$ of their wealth as an bequest to individuals born at the same time t; the remaining fraction $1-\eta$ of this wealth is lost in storage.[4] We write $y(t_0)$ as the (common) inheritance of individuals who are born at time t_0. So an individual of quality Q born

[4]If $\eta = 0$ (no inheritance) or $\eta = 1$ (no loss in bequests), no non-degenerate steady state could exist.

at time t_0 begins life with *wealth* $X_Q(t_0) = y(t_0)$; its wealth changes during its lifetime at the rate:

$$\frac{dX_Q(t)}{dt} = F_Q(t) - 1/2 \tag{3}$$

We stress that an individual's wealth may shrink or grow; if it shrinks, it may eventually shrink to 0 before the individual dies of natural causes in which case the individual dies in poverty. Of course individuals who die in poverty do not leave a bequest. In our analysis, we show that the system has a unique non-degenerate steady state. In this steady state, $dX_0(t)/dt < 0$ and $dX_1(t)/dt > 0$ so the wealth of low quality individuals shrinks and the wealth of high quality individuals grows; it follows that some low quality individuals die in poverty but no high quality individuals die in poverty.

We have defined the state of society at time t in terms of the population distributions $\mathcal{P}_0, \mathcal{P}_1$; however in analyzing the evolution of society it is more convenient to work with the *population densities* p_0, p_1. By definition,

$$\mathcal{P}_Q(x,t) = \int_0^x p_Q(\hat{x},t)\, d\hat{x}$$

Working with densities is more convenient because their evolution is determined by the following evolution equations, which are based on the principle of mass conservation. (See for instance the discussion of the *continuity equations* in Wikipedia.)

$$\begin{aligned}
\frac{\partial p_0(x,t)}{\partial t} &= -\frac{\partial p_0(x,t)}{\partial x}[F_0(t) - 1/2] - \lambda_d\, p_0(x,t) \\
\frac{\partial p_1(x,t)}{\partial t} &= -\frac{\partial p_1(x,t)}{\partial x}[F_1(t) - 1/2] - \lambda_d\, p_1(x,t)
\end{aligned} \tag{4}$$

In these equations, the first term on the right hand side is the flux density arising from the accumulation/de-accumulation of wealth and the second term is the death rate from natural causes. Note that neither deaths in poverty nor births appear in the evolution equations. This is because deaths in poverty only occur at $x = 0$ and births only occur at $x = y(t)$ (inheritance at time t); the equations are not presumed to hold at these values of x. Deaths in poverty and births enter into the behavior of the system as "boundary conditions" at $x = 0, x = y(t)$ (see Appendix A). It is important to keep in mind that these evolution equations are *coupled* because productivity $F_Q(t)$ of agents of quality Q depends on the *average quality of the population* and hence on the *populations of each quality* and not just on the population of quality Q. Note too that the "boundary" $x = y(t)$ is *moving* because inheritance $y(t)$ is a function of the population distributions and hence depends on time.

3 Steady State

As discussed in the Introduction, we are interested in societies in the steady state. We define the *steady state* as the state of the society in which the distribution of individuals (of each type) across wealth levels is unchanging over time; i.e., $\partial p_Q(x,t)/\partial t \equiv 0$ for $Q = 0, 1$. In the steady state, the birth and (overall) death rate are constant and equal, so the populations $P_0(t), P_1(t), P(t)$ are constant; write P_0^s, P_1^s, P^s for the steady state values. Because the population is constant, so are the average quality $Q^s = P_1^s/P^s$, the productivities of individuals of each quality $F_Q^s = zQ + (1-z)Q^s - cP^s$, and inherited wealth Y^s. (All these values will be determined endogenously by the parameters of the model and the condition that the society is in steady state.)

There is always a degenerate steady state in which population is identically 0. In order to guarantee that a non-degenerate steady state exists, we need several assumptions.

Assumptions

1) $\lambda_d < \lambda_b < 2\lambda_d$

2) $\frac{\lambda_d}{\lambda_b} < \gamma$

3) $0 < z < \frac{\lambda_d}{\lambda_b}$

4) $1/(1 + \frac{[1-\lambda_d/\lambda_b]}{\ln(\frac{1}{2[1-\lambda_d/\lambda_b]})}) < \eta < 1$

Some comments on these assumptions are in order. If the natural birth rate were less then the natural death rate then the population of society would shrink to 0 in the long run so the only steady state would be degenerate. Similar reasoning explains the second assumption. To see why the third assumption is needed, suppose for a moment that $z = 0$, so that the society were completely collectivist. In a completely collectivist society, individual output depends only on average quality and not on individual quality, and hence net output (output minus consumption) in a steady state would be $\gamma\bar{Q}^s - cP^s - 1/2$. If net output were positive, inheritance would blow up; if net output were negative, inheritance would shrink to 0. Hence in the steady state, net output must be 0. But this means that no individuals die in poverty; since the natural birth rate is assumed to strictly exceed the natural death rate, this is incompatible with a non-degenerate steady state. Hence a completely collectivist society cannot persist in a non-degenerate steady state. Similar reasoning shows that an extremely individualistic society cannot persist in a non-degenerate steady state; the necessity of the given upper bound is derived in the proof of Theorem 1. Put differently: our model cannot apply to a society that is too collectivistic or too individualistic. The last assumption asserts that some – but not too much – wealth is lost in inheritance. If η were below the given bound then, as the proof of Theorem 1 demonstrates, the population of low quality individuals would go to 0; if η were equal to 1 total wealth would explode. In either case, no non-degenerate steady state could exist.

These four Assumptions will be maintained in what follows without further comment. Given these Assumptions, we can show that there is a unique non-degenerate steady state. We defer the proof of this result (and all others) to Appendix A.

Theorem 1. *There is a unique non-degenerate steady state.*

Figure 1 illustrates the steady state wealth distribution for a particular array of the parameters. Note that the wealth of all High quality individuals is above the inheritance level, that the wealth of all Low quality individuals is below the inheritance level, and that the total population of High quality individuals is greater than that of Low quality individuals.

4 Model Predictions

We now derive various implications for economic outcomes.

To understand what drives these implications, it is useful to think about the various forces at work and how they manifest in the various aspects of the steady state. Throughout the discussion, we take birth and death rates λ_b, λ_d and inheritability η of bequests as fixed, so that the steady state (and all the steady state quantities) are functions of the congestion coefficient c and group productivity γ – which we think of as technological parameters – and the degree of individualism z – which we think of as the cultural parameter.[5] The forces that these parameters generate can be seen most easily by viewing the unique non-degenerate steady state population $P^s(c, \gamma, z)$ as a function of the technological and cultural parameters and asking, in the usual *ceteris paribus* fashion, how the non-degenerate steady state population changes as a single parameter changes. With an obvious abuse of language we may speak of one of these parameters being or becoming larger. Intuitively at least we can reason as follows.

- If we hold group efficiency γ and degree of individualism z fixed then a larger congestion parameter c generates a downward force on the population. To see this, note that a larger c implies a greater (negative) congestion effect, so that productivity will be lower in both individual and group modes. Hence the wealth of low quality individuals will decline more quickly and wealth of high quality individuals will increase more slowly. From this it also follows that individuals who die of natural causes will leave a smaller bequest, and hence that newborn individuals will come into a smaller inheritance. In particular,

[5]In the more general model sketched in Appendix B, we might think of the individual productivities, the subsistence level and the extent of consumption above the subsistence level as technological parameters as well.

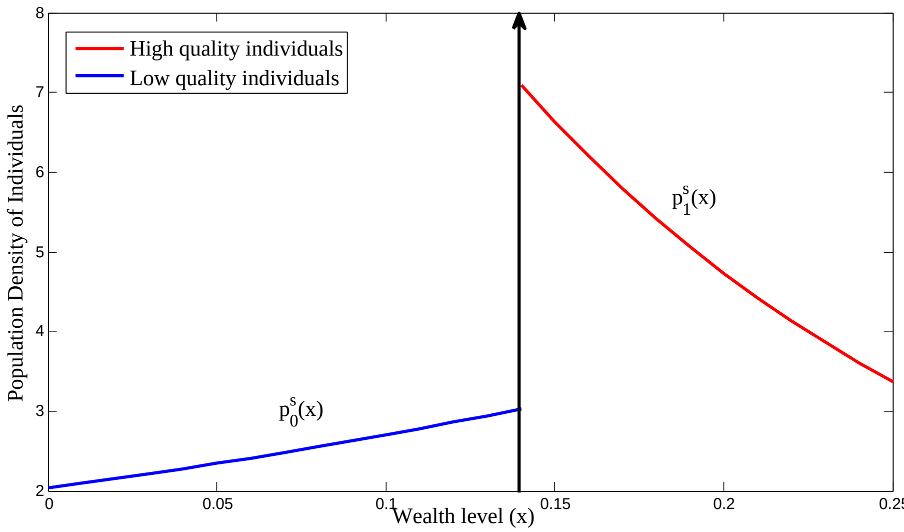

Figure 1: Steady State Distribution of Wealth

low quality individuals will begin with less wealth, spend that wealth faster, and hence be more likely to die in poverty before they die of natural causes. This suggests that a larger congestion parameter should lead to a smaller steady state population.

- If we hold congestion c and degree of individualism z fixed then greater group efficiency γ generates an upward force on the population. To see this note that, holding population fixed, greater group efficiency means greater productivity for both high and low quality individuals when working with others. Hence the wealth of low quality individuals will decline more slowly and the wealth of high quality individuals will increase more quickly. From this, it also follows that individuals who die of natural causes will leave a larger bequest, and hence that newborn individuals will come into a larger inheritance. In particular, low quality individuals will begin with greater wealth, spend that wealth more slowly, and hence be less likely to die in poverty before they die of natural causes. This suggests that greater group efficiency should lead to a smaller steady state population.

- However if we hold congestion c and group efficiency γ fixed then a greater degree of individualism z generates *both* upward *and* downward forces on the population. To see this note that, on the one hand, low quality individuals produce *more* per unit time when working with others than when working alone, so working with others provides low quality workers with a "safety net." A greater degree of individualism lowers this "safety net", so that the wealth of low quality workers more quickly and they die in poverty *more often*. On the other hand (at least if γ is not too large) high quality individuals produce *less* per until time when working with others than when working alone. A greater degree of individualism therefore increases the rate at which high quality workers accumulate wealth, and hence increases the bequests they leave when they die, which in turn implies that low quality individuals begin life with greater wealth and tend to die in poverty *less often*. Evidently, these forces work in opposite directions, which suggests that the overall impact on population depends on the balance between them; we show below, the net effect depends on the relative magnitude of all the parameters.

As Theorem 2 below demonstrates formally, these intuitions about the impact of parameters on steady state population are indeed correct. However, we warn the reader that, as we will see later, apparently similar intuitions about the impact of parameters on other economic outcomes *are not correct*: the technological parameters (the congestion coefficient c and the group productivity γ) do not matter for mean income or for income inequality.

Theorem 2. *In the non-degenerate steady state:*

(a) population is decreasing in the congestion parameter: $\partial P^s/\partial c < 0$

(b) population is increasing in group productivity: $\partial P^s/\partial \gamma > 0$

(c) for each $c > 0$ *there is a threshold* $\gamma^* < 1$ *such that*

- *(i) if* $\gamma < \gamma^*$ *then population is increasing in the degree of individualism:* $\partial P^s/\partial z > 0$
- *(ii) if* $\gamma > \gamma^*$ *then population is decreasing in the degree of individualism:* $\partial P^s/\partial z < 0$

(In both cases, $\partial P^s/\partial z$ *is constant, so* P^s *is linear in* z*.)*

Theorem 2 describes the dependence of the total population on the various parameters but is silent about the dependence of the populations of each quality and the ratio of these populations. Perhaps surprisingly, as Theorem 3 below asserts formally, this ratio is *independent* of all the parameters. To understand the intuition for this conclusion, suppose the parameters change in such a way that the population of low quality workers grows. Because the birth rate and the ratio of low quality births to high quality births are constant, the population of high quality workers must also grow – and, as we show, it will grow at precisely the same rate as the population of low quality workers, so that the ratio of the populations remains constant.

Theorem 3. *In the non-degenerate steady state, the population ratio* P_0^s/P_1^s *is independent of* c, γ, z.

We now turn from population to income, in particular to mean income and to income inequality. We identify income with output so the *per capita income of society* or *GDP per capita* in the steady state is

$$F^s = [F_0^s P_0^s + F_1^s P_1^s]/P^s$$

As with population, we view $F^s = F^s(c, \gamma, z)$ as a function of the technological and cultural parameters.

Theorem 4. *In the non-degenerate steady state:*

(a) GDP per capita is independent of the congestion parameter: $\partial F^s/\partial c = 0$

(b) GDP per capita is independent of group productivity: $\partial F^s/\partial \gamma = 0$

(c) GDP per capita is increasing in the degree of individualism: $\partial F^s/\partial z > 0$

As we have noted in the Introduction, Theorem 4 is consistent with a general view of the Malthusian era that an improvement in technology (in our case, a decrease in the congestion factor c or an increase in the social productivity factor γ) may lead in the short run to a temporary increase in GDP per capita but in the longer run simply to a larger population. (See Ashraf and Galor [3] for data that documents this view.) Our model suggests a mechanism leading to this conclusion: an improvement in technology – either a decrease in the congestion factor c or an increase in the social productivity factor γ – leads to an increase in the population (Theorem 2) which in turn leads to an increase in congestion $cP(t)$, which in term reduces (per capita) gains to the improved technology; in the steady state, these forces exactly balance out. (Ashraf and Galor [3] offer a very different model that leads to the same conclusion.)

We measure income inequality in the familiar way as the Gini coefficient of the income distribution. Because there are only two types of individuals, the Gini coefficient takes the particularly simple form

$$G^s = F_1^s P_1^s / F^s P^s - P_1^s / P^s = \left[\frac{P_1^s}{P^s}\right]\left[\frac{F_1^s}{F^s} - 1\right]$$

As before, we view $G^s = G^s(c, \gamma, z)$ as a function of the technological and cultural parameters.

Theorem 5. *In the non-degenerate steady state:*

(a) the Gini coefficient is independent of the congestion parameter: $\partial G^s / \partial c = 0$

(b) the Gini coefficient is independent of group productivity: $\partial G^s / \partial \gamma = 0$

(c) the Gini coefficient is increasing in the degree of individualism: $\partial G^s / \partial z > 0$

5 Historical Evidence

As we have said before, we intend our model to be descriptive of societies in the Malthusian Era, the period between the Neolithic Revolution and the Industrial Revolution. Although only a limited amount of data is available for this period and there is some disagreement about its quality, it nevertheless seems appropriate to compare the predictions of our model with the data that is available.

Our model makes use of a number of parameters: the birth and death rates λ_b, λ_d, the fraction η of wealth that is inheritable, the coefficient c of congestion, the group efficiency γ, and the degree z of individualism. Unfortunately, none of these parameters can be observed directly. (At least, none of these parameters were observed directly in the data that is available to us.) What *is* available is an index of individualism calculated by Hofstede [12], which we use as a proxy for z (rescaled to lie in $[0, 1]$).[6] In comparing

[6] A natural alternative would be to assume that z is a (monotone) Box-Cox transformation [4] of Hofstede's index. We have in fact computed the optimal Box-Cox transformation and carried through the regressions after performing the optimal Box-Cox transformation; however, there is almost no change in either the regression lines or the fit to the data. The results of these regressions are available from the authors on request.

the predictions of our model with historical data we make the simple (but perhaps heroic) assumption that birth and death rates and the fraction of wealth that is inheritable are the same across societies. It seems completely implausible to assume that technologies are the same across societies – and hence that the technological parameters c, γ are the same across societies – so we focus on the predictions for mean income and Gini coefficient, which are independent of these parameters.

To examine the implications of Theorem 4 with historical data, we use estimates of GDP in 1500 CE provided in [15] for Western Europe. We identify mean income with GDP per capita We use linear least-squares regression to compute the best-fitting straight line; see Figure 2. (Note that some of the "countries" that appear in Figure 2 – e.g. Italy – did not exist in 1500. Maddison uses the names to refer to the geographic areas occupied by the *current* countries.) Note that $R^2 = .25$ so that the theory explains a significant amount of the data.

Unfortunately, we do not find any data for Gini coefficients from 1500 CE (the period of the data used above). We therefore use the estimates of Gini coefficients given by Williamson et. al. [17] from the (roughly) 100 year period 1788-1886 C.E., which might be thought to be after the Industrial Revolution. However, for those countries in which the Industrial Revolution arrived early (especially England, France and The Netherlands), the data and the calculations/estimations are from the beginning of this period, which would seem to be (mostly) *before* the (full impact of the) Industrial Revolution, while for those countries (especially Brazil, China and Peru) for which the data and the calculations/estimations are from the end of this period, the Industrial Revolution did not in fact arrive until much later. The data and regression results can be seen in Figure 3. Note that $R^2 = .63$ so that the theory explains quite a bit of the data.

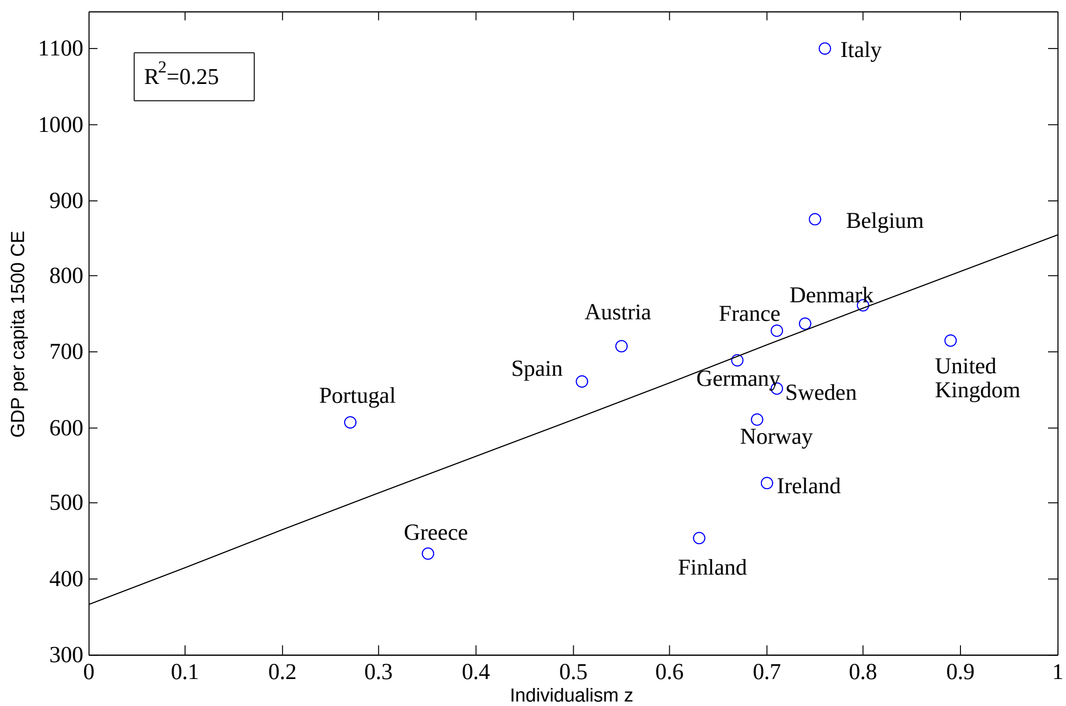

Figure 2: Mean Income (GDP per capita) vs. Individualism

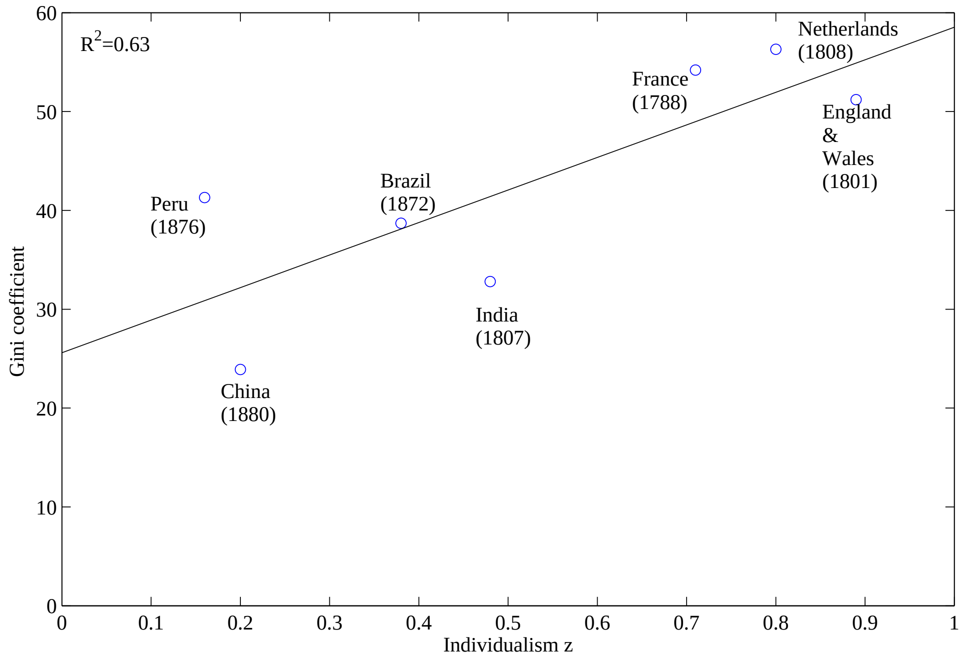

Figure 3: Gini Coefficients vs. Individualism

6 Discussion and Conclusions

This paper proposes and analyzes a model that provides a mechanism by which the tension between individualism and collectivism can lead to different economic outcomes in different societies. The model captures important features of the period between the Neolithic Revolution and the Industrial Revolution era as discussed in the work of Clark [5] and others. The model makes predictions about the impact of individualism and collectivism on different societies, and these predictions seem consistent with (very limited) historical data.

We reach no conclusion as to whether individualism or collectivism is "better" – indeed, the predictions of the model show that such a conclusion would depend entirely on the criteria used. In particular, our prediction is that a greater degree of individualism leads to higher mean income (GDP per capita) but also to greater inequality.

The model presented above makes many simplifying assumptions – but the model could be generalized in many dimensions (generalizing individual productivities and subsistence consumption, and allowing for non-linear congestion and non-constant fertility and death rates, for instance) without qualitative changes in the conclusions. Other generalizations might allow for the possibility that individual output and deaths due to poverty are stochastic (rather than deterministic) – but such generalizations would seem to lead to enormous complications. Perhaps the most important simplifying assumption that has not been discussed is that all newborns inherit an equal share of bequests. Many modifications of this assumption seem possible (e.g. that children inherit only from their parents) but many of those modifications seem to lead to technical challenges in formulating and/or solving the model.

We have confined our analysis to the non-degenerate steady state of the society which seems reasonable given that we are interested in the Malthusian Era in which there was little or no change. However even in the Malthusian era there were shocks – famines and epidemics – which perturbed the society from its steady state, so it would certainly be of interest to understand the stability properties of the model. From which perturbations would a society in the (non-degenerate) steady state return to the (non-degenerate) steady state? From which perturbations would the system collapse to the degenerate steady state? (As Diamond [7] documents, such collapses are far from unknown.) Unfortunately, this an extremely complicated problem and well beyond our capabilities. Out of the steady state the dynamics of our model are governed by coupled PDE's with a moving boundary condition, and so the future evolution of the system depends on the entire wealth distribution and not just on a few aggregates. Such dynamics are well-known to be extremely resistant to rigorous analysis – or indeed, even to simulation, which are often extremely sensitive to the precise small details of the numerical approximation.

Appendix A

Here we present the proofs for the formal results discussed in the text. Before we being, recall that the productivity of an individual of quality Q at time t is:

$$F_Q(t) = z[Q - cP(t)] + (1-z)[\bar{Q}(t)\gamma - cP(t)] = zQ + (1-z)\gamma\bar{Q}(t) - cP(t)$$

Note low quality individuals are always more productive when working collectively, but whether high quality individuals are more or less productive when working collectively depends on whether $\gamma\bar{Q}(t) > 1$ or $\gamma\bar{Q}(t) < 1$, and this is determined endogenously.

Proof of Theorem 1 Since the proof is a bit roundabout, it may be useful to begin with an overview. By definition, a steady state is a pair of density functions $p_0(x,t), p_1(x,t)$ that satisfy the evolution equations and are independent of time t. In the steady state, the populations P_0^s, P_1^s and inheritance Y^s are constant, so average quality Q^s and productivities F_0^s, F_1^s are also constant. Hence we can identify a steady state as a pair of functions $p_0(x), p_1(x)$ that satisfy the steady state evolution equations

$$\frac{\partial p_0(x)}{\partial x}[F_0^s - 1/2] = -\lambda_d p_0(x) \tag{SSEE0}$$

$$\frac{\partial p_1(x)}{\partial x}[F_1^s - 1/2] = -\lambda_d p_1(x) \tag{SSEE1}$$

and also satisfy the appropriate boundary conditions. Note that, although we have written these as partial differential equations, because the functions $p_0(x), p_1(x)$ are independent of time (which is what it means to be in the steady state), they are reduced to *ordinary* differential equations. Of course they remain *coupled*, because steady state productivities F_0^s, F_1^s depend on the entire population and not just on the population of each quality separately.

We therefore begin with candidate steady state populations P_0^s, P_1^s and inheritance Y^s (satisfying some conditions that must hold in any steady state of the system). For any such triple, we show that the equations SSEE0, SSEE1 admit unique solutions which yield the given steady state quantities. We then show that the boundary conditions uniquely pin down the *unique* triple of steady state quantities that correspond to an actual steady state *of the society.*

We begin by considering any non-degenerate solution $p_0^s(x), p_1^s(x)$ to the steady state evolution equations (not necessarily satisfying boundary conditions). From these, we can derive the following steady state quantities:

- the population of individuals with quality Q

$$P_Q^s = \int_0^\infty p_Q^s(x)dx \tag{5}$$

- the total population

$$P^s = P_1^s + P_0^s = \int_0^\infty [p_1^s(x) + p_0^s(x)]dx \tag{6}$$

- mean quality

$$Q^s = P_1^s/(P_0^s + P_1^s) \tag{7}$$

- productivity of individuals of quality Q

$$F_Q^s = zQ + (1-z)\gamma Q^s - cP^s \tag{8}$$

- mean wealth

$$X^s = \frac{\int_0^\infty x[p_1^s(x) + p_0^s(x)]dx}{\int_0^\infty [p_1^s(x) + p_0^s(x)]dx} \tag{9}$$

- inheritance

$$Y^s = \lambda_d P^s X^s \eta / \lambda_b P^s = (\lambda_d/\lambda_b) X^s \eta \tag{10}$$

Because we have assumed a non-degenerate steady state we must have $P^s \neq 0$ so $P_0^s \neq 0$ and $P_1^s \neq 0$. Note that the three quantities P_0^s, P_1^s, Y^s determine all the others.

We assert that in a non-degenerate steady state we must have

$$F_0^s < 1/2 < F_1^s$$

so that low quality individuals produce less than they consume and high quality individuals produce less than they consume. To show this we show that the other possibilities are incompatible with a non-degenerate steady state. Note first of all that the definitions and the assumption that $0 < z < 1$ imply that $F_0^s < F_1^s$ so we must rule out the only two other possibilities:

- $1/2 \leq F_0^s < F_1^s$. If this were the case then the wealth of low quality individuals would be non-decreasing during their lifetimes and the wealth of high quality individuals would be strictly increasing during their lifetimes, so social wealth would be strictly increasing, which is impossible in the steady state.
- $F_0^s < F_1^s \leq 1/2$. If this were case then the wealth of low quality individuals would be strictly decreasing and the wealth of high quality individuals would be non-increasing, so social wealth would be strictly decreasing, which is impossible in the steady state.

We therefore conclude that $F_0^s < 1/2 < F_1^s$ as asserted.

In order to show that a non-degenerate steady state of the society exists and is unique we proceed in the following way. We have shown that, beginning with a solution p_0^s, p_1^s to the steady state evolution equations (SSEE0), (SSEE1) we can derive a triple of steady state quantities P_0^s, P_1^s, Y^s having the property that $F_0^s < 1/2 < F_1^s$. The first part of the proof is to show that, for every such triple of steady state quantities there is a unique solution p_0^s, p_1^s to the steady state evolution equations that yields the given steady state quantities. The second part of the proof is to show that the boundary conditions uniquely pin down the triple of steady state quantities that correspond to an actual steady state of the society.

To this end, fix a triple of steady state quantities P_0^s, P_1^s, Y^s for which total population is positive $P_1^s + P_0^s = P^s > 0$, inheritance is non-negative $Y^s \geq 0$ and for which the derived quantities F_0^s, F_1^s satisfy $F_0^s < 1/2 < F_1^s$. In any solution of the steady state evolution equations that yields these steady state quantities, it is by true by definition that all individuals are born with the inheritance Y^s. Because $F_0^s < 1/2 < F_1^s$, the wealth of low quality individuals is strictly decreasing while they are alive and the wealth of high quality individuals is strictly increasing while they are alive. Hence, $p_0^s(x) = 0$ for $x > Y^s$ and $p_1^s(x) = 0$ for $x < Y^s$; equivalently, p_0^s is supported on $[0, Y^s]$ and p_1^s is supported on $[Y^s, \infty)$. From these facts we can determine the desired population distributions p_0^s and p_1^s.

To determine p_1^s, set $\lambda_1 = \lambda_d/[F_1^s - 1/2]$. For $x > Y^s$, the function p_1^s solves the ODE:

$$\frac{dp_1^s(x)}{dx} = -\lambda_1 p_1^s(x) \tag{11}$$

The solution to this ODE is of the form

$$p_1^s(x) = C_1 e^{-\lambda_1(x-Y^s)} \tag{12}$$

where the multiplicative constant C_1 is determined by initial conditions. Given p_1^s we find that $P_1^s = C_1/\lambda_1$ so that

$$p_1^s(x) = \begin{cases} P_1^s \lambda_1 e^{-\lambda_1(x-Y^s)} & \text{for } x > Y^s \\ 0 & \text{for } x < Y^s \end{cases} \tag{13}$$

Note that $\lambda_1 = \lambda_d/[F_1^s - 1/2]$ and recall that F_1^s can be expressed in terms of P_1^s, P_0^s.

To determine p_0^s, set $\lambda_0 = -\lambda_d/[F_0^s - 1/2]$. For $x < Y^s$ the function p_0^s satisfies the ODE:

$$\frac{dp_0^s(x)}{dx} = \lambda_0 p_0^s(x) \tag{14}$$

The solution to this ODE is of the form

$$p_0^s(x) = C_0 e^{\lambda_0(x-Y^s)} \tag{15}$$

where the multiplicative constant C_0 is determined by initial conditions. Given p_0^s we find that $P_0^s = (C_0/\lambda_0)(1 - e^{-\lambda_0 Y^s})$ so that

$$p_0^s(x) = \begin{cases} \left[P_0^s \lambda_0/(1 - e^{-\lambda_0 Y^s})\right] e^{\lambda_0(x-Y^s)} & \text{for } x < Y^s \\ 0 & \text{for } x > Y^s \end{cases} \tag{16}$$

Note that $\lambda_0 = -\lambda_d/[F_0^s - 1/2]$ and recall that F_0^s can be expressed in terms of P_1^s, P_0^s.

By construction, the functions p_0^s, p_1^s satisfy the steady state evolution equations. Direct calculation shows that the steady state quantities derived from p_0^s, p_1^s are precisely the quantities P_0^s, P_1^s, Y^s with which we began. This completes the first part of the proof.

We now turn to the second part of the proof which is to pin down the steady state values of P_1^s, P_0^s, Y^s that correspond to the (unique) non-degenerate steady state of the society.

Note first that because half of newborns are of low quality and half are of high quality, we have the following boundary condition:

$$\lim_{x \downarrow Y^s} p_1^s(x)|F_1^s - 1/2| = \lim_{x \uparrow Y^s} p_0^s(x)|F_0^s - 1/2| \tag{17}$$

(As usual, $\lim_{x \downarrow Y^s}$ is the limit from the right and $\lim_{x \uparrow Y^s}$ is the limit from the left.) Simplifying yields

$$P_1^s = \frac{P_0^s}{1 - e^{-\lambda_0 Y^s}} \tag{18}$$

and hence that

$$e^{-\lambda_0 Y^s} = (2 - P^s/P_1^s) \tag{19}$$

Next we compute the rate μ^s at which individuals die in poverty in the steady state. (Of course, only low quality individuals die in poverty.)

$$\begin{aligned} \mu^s &= f_0^s(0)|F_0^s - 1/2| \\ &= C_0 e^{-\lambda_0 Y^s}|F_0^s - 1/2| \\ &= (C_0/\lambda_0)\lambda_d e^{-\lambda_0 Y^s} \\ &= P_1^s \lambda_d e^{-\lambda_0 Y^s} \\ &= (2P_1^s - P^s)\lambda_d \end{aligned} \tag{20}$$

In the steady state the population is constant so the birth rate must equal to death rate, which yields the second boundary condition:

$$(\lambda_b P^s - \lambda_d P^s - \mu^s) = 0 \tag{21}$$

Substituting gives:

$$\lambda_b P^s - \lambda_d P^s - \lambda_d(2P_1^s - P^s) = 0 \tag{22}$$

Hence, we have

$$P_1^s = \lambda_b/(2\lambda_d)P^s \tag{23}$$

By assumption, $\eta < 1$ is the fraction of wealth that is transferred across generations so:

$$\lambda_b Y^s = \eta \lambda_d X^s \tag{24}$$

Next we compute X^s.

$$X^s = \frac{1}{2 - e^{-\lambda_0 Y^s}} \Big[\int_0^{Y^s} \lambda_0 x e^{\lambda_0 (x - Y^s)} dx + \int_{Y^s}^{\infty} \lambda_1 x e^{-\lambda_1 (x - Y^s)} dx \Big] \tag{25}$$

Integration by parts yields:

$$\begin{aligned} \int_0^{Y^s} \lambda_0 x e^{\lambda_0 (x - Y^s)} dx &= \Big[\frac{e^{-\lambda_0 Y^s} - 1}{\lambda_0} \\ &+ \quad \frac{\lambda_0 Y^s e^{-\lambda_0 Y^s}}{\lambda_0} + Y^s - Y^s e^{-\lambda_0 Y^s} \Big] \\ \int_{Y^s}^{\infty} \lambda_1 x e^{-\lambda_1 (x - Y^s)} dx &= \frac{1 + \lambda_1 Y^s}{\lambda_1} \end{aligned} \tag{26}$$

We use the above expressions to simplify X^s:

$$X^s = 2\frac{P_1^s}{P^s} Y^s + \frac{1}{\lambda_d} (\frac{P_1^s}{P_s} [z + (1 - z)\gamma]) - \frac{1}{\lambda_d} (cP^s + 1/2) \tag{27}$$

We can substitute $\frac{P_1^s}{P^s}$ from (23) in the above and substitute X^s from (24) to obtain

$$Y^s = \left(\frac{\lambda_b}{2\lambda_d} [z + (1 - z)\gamma] - cP^s - 1/2 \right) \left(\frac{\eta}{\lambda_b (1 - \eta)} \right) \tag{28}$$

Using the equations (28), (23) and (19) we will determine each of the desired quantities. We write (19) as follows.

$$e^{-\lambda_0 Y^s} = 2 - P^s / P_1^s \tag{29}$$

Substitute (23) and the expression for λ_0 in the above and then take logarithms to obtain:

$$\lambda_d Y^s = \ln \left[2 - \frac{2\lambda_d}{\lambda_b} \right] \left[(1 - z)\gamma \frac{\lambda_b}{2\lambda_d} - cP^s - 1/2) \right] \tag{30}$$

Substitute $cP^s + 1/2$ from (28) in the above to obtain:

$$\lambda_d Y^s = \ln\left[2 - \frac{2\lambda_d}{\lambda_b}\right]\left((1-z)\gamma\frac{\lambda_b}{2\lambda_d} + \left[\frac{\lambda_b(1-\eta)}{\eta}\right]Y^s - \frac{\lambda_b}{2\lambda_d}(z+(1-z)\gamma)\right) \tag{31}$$

We can simplify the above to obtain the final expression for Y^s:

$$Y^s = \frac{\eta z}{2\lambda_d(1-\eta+\beta)} \tag{32}$$

where $\beta = -[\eta\,\lambda_d/\lambda_b]/\ln[2 - \frac{2\lambda_d}{\lambda_b}]$. In a non-degenerate steady state we must have $X^s > 0$. We know $X^s = \frac{\lambda_b Y^s}{\lambda_d \eta}$; since $\lambda_d < \lambda_b < 2\lambda_d$ it follows that $2(1-\frac{\lambda_d}{\lambda_b}) \in (0,1)$ and hence that $(1-\eta+\beta) > 0$ and that $X^s > 0$ as required.

Now we substitute (32) in (28) to obtain the expression for P^s as follows.

$$cP^s = \frac{\lambda_b}{2\lambda_d}\left[\gamma + z\left(\frac{\beta}{1-\eta+\beta} - \gamma\right)\right] - \frac{1}{2} \tag{33}$$

Since $\lambda_b > \frac{\lambda_d}{\gamma}$ the above expression is greater than zero when $z = 0$ and since $\eta > 1/(1 + \frac{[1-\lambda_d/\lambda_b]}{\ln(\frac{1}{2[1-\lambda_d/\lambda_b]})})$ the above expression is greater than zero when $z = 1$. This ensures that $P^s > 0$. We know that

$$P_1^s = \left[\frac{\lambda_b}{2c\lambda_d}\right]\left[\frac{\lambda_b}{2\lambda_d}\left[\gamma + z\left(\frac{\beta}{1-\eta+\beta} - \gamma\right)\right] - \frac{1}{2}\right] \tag{34}$$

and

$$P_0^s = \left[\frac{1}{c} - \frac{\lambda_b}{2c\lambda_d}\right]\left[\frac{\lambda_b}{2\lambda_d}\left[\gamma + z\left(\frac{\beta}{1-\eta+\beta} - \gamma\right)\right] - \frac{1}{2}\right] \tag{35}$$

Since $P^s > 0$ both P_1^s and P_0^s are greater than zero. This derivation was based on the assumption that $F_0^s < 1/2 < F_1^s$; we now check that this is indeed true for the derived values of P_0^s, P_1^s, X^s.

We treat F_1^s first. Substitute (33) to obtain

$$\begin{aligned} F_1^s - 1/2 &= z + (1-z)\gamma\frac{\lambda_b}{2\lambda_d} - cP^s - 1/2 \\ &= z(1 - \frac{\lambda_b}{2\lambda_d}) + \frac{z\lambda_b(1-\eta)}{2\lambda_d(1-\eta+\beta)} \end{aligned} \tag{36}$$

Because $z > 0$ and $\lambda_b < 2\lambda_d$ the first term in the right hand side is strictly positive. Because $(1 - \frac{\lambda_b}{2\lambda_d}) > 0$ and $(1 - \eta + \beta) > 0$ the second term is strictly positive, so $F_1^s - 1/2 > 0$.

We now turn to F_0^s. Substitute (33) to obtain

$$\begin{aligned} F_0^s - 1/2 &= (1-z)\gamma \frac{\lambda_b}{2\lambda_d} - cP^s - 1/2 \\ &= z \left[\frac{\lambda_b}{2\lambda_d}\right] \left[\frac{-\beta}{1-\eta+\beta}\right] \end{aligned} \tag{37}$$

Since $z > 0$, $\beta > 0$ and $(1 - \eta + \beta) > 0$, we conclude that $F_0^s - 1/2 < 0$. To see that $F_0^s > 0$ we calculate:

Now we have determined the values of P_1^s, P_0^s, Y^s in (34), (35) and (32). We can substitute these in (13) and (16) to obtain the final distribution function. This completes the proof.

Proof of Theorem 2 In the proof of Theorem 1 we arrived at an expression for cP^s in (33), so we conclude that

$$P^s = \left(\frac{1}{c}\right)\left(\frac{\lambda_b}{2\lambda_d}\left[\gamma + z\left(\frac{\beta}{1-\eta+\beta} - \gamma\right)\right] - \frac{1}{2}\right) \tag{38}$$

where $\beta = -(\eta\,\lambda_d/\lambda_b)/\ln(2 - \frac{2\lambda_d}{\lambda_b})$. It is immediate that P^s is decreasing in c and increasing in γ. P^s is evidently linear in z; P^s is decreasing in z if $\gamma > \frac{\beta}{1-\eta+\beta}$ and is increasing in z if $\gamma < \frac{\beta}{1-\eta+\beta}$, so that $\gamma^* = \frac{\beta}{1-\eta+\beta}$ is the asserted threshold. This completes the proof.

Proof of Theorem 3 In the proof of Theorem 1, we arrived at equation (23) which expresses the population P_1^s of high quality individuals as a fraction of the total population P^s. Since $P^s = P_0^s + P_1^s$, simple algebra shows that the steady state population ratio is

$$\frac{P_0^s}{P_1^s} = 1 - \frac{\lambda_b}{2\lambda_d} \tag{39}$$

Note that Assumption 1 guarantees that the right hand side is strictly positive and less than 1. This completes the proof.

Proof of Theorem 4 We first derive the expression for mean income. We know that $P^s_1/P^s = Q^s = \frac{\lambda_b}{2\lambda_d}$. In the simplification given below we use the expression derived in (36) and (37).

$$F^s = Q^s F^s_1 + [1 - Q^s]\, F^s_0 \tag{40}$$

$$F^s = \frac{(1-\eta)z}{1-\eta+\beta} + \frac{1}{2} \tag{41}$$

Note that mean income F^s is independent of the technological parameters c, γ and linear in the degree of individualism z. Because $\eta < 1$, mean income is increasing in the degree of individualism z.

Proof of Theorem 5 We have seen in the proof of Theorem 1 that both income levels are positive, so writing $A = \frac{(1-\eta)}{(1-\eta+\beta)}$ and performing the requisite algebra yields a convenient expression for the Gini coefficient is:

$$\begin{aligned} Gini &= \frac{Q^s F^s_1}{Q^s F^s_1 + (1-Q^s)\, F^s_0} - Q^s \\ &= Q^s \left[\frac{(1-Q^s)F^s_1 - (1-Q^s)F^s_0}{F^s} \right] \\ &= Q^s(1-Q^s)\left(\frac{F^s_1 - F^s_0}{F^s} \right) \\ &= Q^s(1-Q^s)\left(\frac{z}{Az + 1/2} \right) \end{aligned} \tag{42}$$

Because the steady state average quality Q^s depends only on the steady state population ratio P^s_0/P^s_1, which is independent of the technological parameters c, γ, we see that the Gini coefficient is also independent of the technological parameters c, γ.

Finally, differentiating the expression for the Gini coefficient yields

$$\frac{\partial Gini}{\partial z} = \frac{\lambda_b}{2\lambda_d}\left[1 - \frac{\lambda_b}{2\lambda_d}\right]\left[\frac{1}{2(Az+1/2)^2}\right] \tag{43}$$

Since $\lambda_b < 2\lambda_d$ the Gini coefficient is increasing in the level of individualism z, as asserted.

Appendix B

Here we sketch the more general version of the model promised in the main text.

We assume that, when working alone, individuals of quality Q produce at the constant rate π_Q, where $0 \leq \pi_0 < \pi_1$. All individuals must consume at least a subsistence level σ; we assume $\pi_0 < \sigma < \pi_1$. Individuals who produce less than σ consume part of their wealth; individuals who produce more than σ consume a fraction β of the excess. Hence production of an individual of quality Q is $F_Q(t) = z[\pi_Q - cP(t)] + (1-z)[\gamma\bar{Q}(t) - cP(t)]$ and production net of consumption is $F_Q(t) - \sigma - \beta[F_Q(t) - \sigma]^+$. The continuity equations become

$$\begin{aligned}
\frac{\partial p_0(x,t)}{\partial t} &= -\frac{\partial p_0(x,t)}{\partial x}\Big(F_0(t) - \sigma - \beta[F_0(t) - \sigma]^+\Big) - \lambda_d\, p_0(x,t) \\
\frac{\partial p_1(x,t)}{\partial t} &= -\frac{\partial p_1(x,t)}{\partial x}\Big(F_1(t) - \sigma - \beta[F_1(t) - \sigma]^+\Big) - \lambda_d\, p_1(x,t)
\end{aligned}$$

In the steady state, it is necessarily the case that low quality individuals produce less than they consume and high quality individuals produce more than they consume (else the population would grow) so

$$\begin{aligned}
F_0^s - \sigma - \beta[F_0^s - \sigma]^+ &= F_0^s - \sigma \\
F_1^s - \sigma - \beta[F_1^s - \sigma]^+ &= (1-\beta)[F_1^s - \sigma]
\end{aligned}$$

Hence the steady state evolution equations become

$$\begin{aligned}
\frac{\partial p_0(x)}{\partial x}[F_0^s - \sigma] &= -\lambda_d p_0(x) \\
\frac{\partial p_1(x)}{\partial x}(1-\beta)[F_1^s - \sigma] &= -\lambda_d p_1(x)
\end{aligned}$$

We can then solve for the steady state exactly as before and all the Theorems will remain valid. Of course the magnitudes of various derivatives will depend on the parameters – but not the signs.

References

[1] Ashraf Q. and Galor, O. (2011), "Dynamics and Stagnation in the Malthusian Epoch," *American Economic Economic Review* 101(5):2003-2041.

[2] Benabou, R., Ticchi, D, and Vindigni, A. (2014), "Forbidden Fruits: The Political Economy of Science, Religion, and Growth," Princeton University Working Paper.

[3] Benabou, R., Ticchi, D, and Vindigni, A. (2015), "Religion and Innovation," *American Economic Review Papers and Proceedings*, 105(5): 346-351.

[4] Box, G. and Cox, D. (1964), "An Analysis of Transformations," *Journal of the Royal Statistical Society, Series B Statistical Methodology* 26(2): 211-252.

[5] Clark, G. (2008), *A Farewell to Alms: A Brief Economic History of the World*, (Princeton Univ Press, Princeton).

[6] Clark, G. (2007), "The Long March of History: Farm Wages, Population, and Economic Growth," *Economic History Review* 60(1): 97-135

[7] Diamond, J. (2005), *Collapse: How Societies Choose to Fail or Survive*, (Penguin Books, New York).

[8] Galor, O. (2011), *Unified growth theory*, (Princeton Univ Press, Princeton).

[9] Greif, A. (1994) "Cultural Beliefs and the Organization of Society: A Historical and Theoretical Reflection on Collectivist and Individualist Societies," *Journal of Political Economy* 102(5): 912-950.

[10] Gorodnichenko, Y. and Roland, G. (2011), "Which Dimensions of Culture Matter for Long-run Growth?" *American Economic Review* 101(3):492-498.

[11] Gorodnichenko, Y. and Roland, G. (2011), "Individualism, Innovation, and Long-run Growth," *Proceedings of the National Academy of Sciences, USA* 108(4):21366-21319.

[12] Hofstede, G. (1984), *Culture's Consequences: International Differences in Work-related Values*, Sage, Beverly Hills, CA.

[13] Landes, D.S. (1999) *The Wealth and Poverty of Nations: Why Some are so Rich and Some so Poor*, WW Norton & Company, New York.

[14] Liebbrant, A., Gneezy, U. and List, J.A. (2013), "Rise and Fall of Competitiveness in Individualistic and Collectivistic Societies," *Proceedings of the National Academy of Sciences, USA* 110(23): 9305-9308.

[15] Maddison, A. (2007), *The World Economy Volume 1: A Millennial Perspective; Volume 2: Historical Statistics*, Academic Foundation.

[16] McEvedy, C. and Jones, R. (1978), *Atlas of World Population History*, Penguin Books Ltd, Harmondsworth, Middlesex, England.

[17] Peter, L.H. and Williamson, J.G. (2007), *Measuring Ancient Inequality*, World Bank Publications.

[18] Voigtlander, N. and Voth, H.J. (2006), "Why England? Demographic Factors, Structural Change and Physical Capital Accumulation During the Industrial Revolution," *Journal of Economic Growth* 11: 319-361.

[19] Voigtlander, N. and Voth, H.J. (2012), "The Three Horsemen of Riches: Plague, War and Urbanization in Early Modern Europe," *Review of Economic Studies* 80(2): 774-811