A Micro-foundation of Social Capital in Evolving Social Networks

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Abstract—A social network confers benefits and advantages on individuals (and on groups); the literature refers to these benefits and advantages as *social capital*. An individual's social capital depends on its position in the network and on the shape of the network – but positions in the network and the shape of the network are determined endogenously and change as the network forms and evolves. This paper presents a micro-founded mathematical model of the evolution of a social network and of the social capital of individuals within the network. The evolution of the network and of social capital are driven by exogenous and endogenous processes – entry, meeting, linking – that have both random and deterministic components. These processes are influenced by the extent to which individuals are *homophilic* (prefer others of their own type), *structurally opportunistic* (prefer neighbors of neighbors to strangers), *socially gregarious* (desire more or fewer connections) and by the *distribution of types* in the society. In the analysis, we identify different kinds of social capital: *bonding capital* refers to links to others; *popularity capital* refers to links from others; *bridging capital* refers to connections between others. We show that each form of capital plays a different role and is affected differently by the characteristics of the society. Bonding capital is created by forming a circle of connections; homophily increases bonding capital because it makes this circle of connections more homogeneous. Popularity capital leads to *preferential attachment*: individuals who become popular tend to become more and more popular because others are more likely to link to them. Homophily creates inequality in the popularity capital attained by different social categories; more gregarious types of agents are more likely to become popular. However, in homophilic societies, individuals who belong to less gregarious, less opportunistic, or major types are likely to be more *central* in the network and thus acquire a bridging capital. And, while extreme homophily maximizes an individual's bonding capital, it also creates *structural holes* in the network, which hinder the exchange of ideas and information across social categories. Such structural holes represent a potential source of bridging capital: non-homophilic (tolerant or open-minded) individuals can fill these holes and broker interactions at the interface between different social categories.

Index Terms—Centrality, homophily, network formation, popularity, preferential attachment, social capital, social networks.

1 Introduction

Social networks bestow benefits – tangible benefits such as physical and monetary resources and intangible benefits such as social support, solidarity, influence, information, expertise, popularity, companionship and shared activity – on the individuals and groups who belong to the network. Such resources allow individuals to do better in the network; they help individuals accomplish tasks, produce and spread information, broker interactions across social categories, display influence on other individuals, gain more knowledge, or enjoy more emotional and social support. The concept of *social capital* has come to embody a set of different incarnations of the benefits attained by social categories via networked societal interactions [1]- [6].

Contemporary sociologists have established different definitions and conceptualizations for social capital. For instance, Coleman has defined the social capital as *"a function of social structure producing advantage"* [1], and he advanced social capital as a conceptual tool that puts economic rationality into a social context [1] [2]. Social capital for Bourdieu is related to the size of network and the volume of past accumulated social capital commanded by an individual [3]. Bourdieu considers that clear profit is the main reason for an individual to engage in and maintain links in a network, and the individuals' potential for accruing social profit and control of capital are non-uniformly distributed. Both conceptualizations of Coleman and Bourdieu are related; they view social capital as existing in relationships and ties, and they postulate that *density* and *closure* are distinctive advantages of capital. While such vision assumes that *strong ties* (the links between homogeneous and like-minded individuals) are the prominent sources of social capital, other sociologists such as Granovetter, Putnam, and Burt have argued that *weak ties* (the links between diverse and weakly connected network components) are also a source of capital [6]- [9]. That is, individuals who can broker connections between otherwise disconnected social categories are more likely to connect non-redundant sources of information, thus promoting for innovation and new ideas [8]. In [8], Burt provided a generalized framework for social capital, viewing bonding capital in connected communities as a source for bridging capital for individuals who connect these communities.

As it is for other forms of capital, inequality is displayed in the creation of social capital [10]; that is to say, social

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capital accrues over time as networks emerge and evolve, and since individuals gain different social positions in the emergent network, capital is not created uniformly across agents; "better connected" agents possess more capital. While there are a number of somewhat different definitions of social capital in the literature, these definitions share the following set of features. First, social capital is a metaphor about advantage, and it can be thought of as the contextual complement of human capital; it is not depleted by use, but rather depleted by non-use. Second, social capital is a function of the collective social structure, and the social positions of individuals; well connected individuals possess more capital, and well connected networks possess a larger shared value. Finally, the creation of social capital exhibits inequality due to the heterogeneity of norms and behaviors of the different social categories, which reflects on their positions in the network.

Motivated by this discussion, this paper aims at establishing the micro-foundations of emerging social capital in an evolving network. In particular, we present a comprehensive mathematical model for dynamic network formation, where agents belonging to heterogeneous social categories take link formation decisions (e.g. "follow" a user on Twitter or ResearchGate [17] [18], "cite" a paper that is indexed by Google Scholar, etc) which on one hand gives rise to an endogenously formed network, and on the other hand creates social capital for individual agents and groups. We view social capital as: *"any advantage or asset that is accrued by an individual or a social category in an evolving network due to the social position that they hold in the underlying network structure. An advantage can correspond to the extent of popularity, prestige, or centrality of an individual; or the density and quality of an individual's ego network."* In our model, we consider that *homophily*, which is an individual's tendency to connect to similar individuals [11], contextualizes economic rationality, i.e. homophily is what creates the incentives for individuals to connect to each other. However, the way individuals meet, the number of links they form, and the way trust propagates among them is governed by norms and behaviors, which generally vary from one social category to another. We view the different forms of social capital as being emergent by virtue of an evolving network, where the evolution of the network is highly influenced by both the actions of individuals, as well as the norms and behaviors of social categories. Due to the heterogeneity of the norms and behaviors of different social categories, social capital inequality is exhibited, and some social categories would collectively acquire more prominent positions in the network than others. In the following subsection, we briefly describe the basic elements of our model.

1.1 A Micro-foundational Perspective of Network Evolution and Social Capital Emergence

The central goal of the paper is to study the micro-foundations of different forms of emerging social capital via a mathematical model for network evolution. In our model, networks are formed over time by the actions of boundedly rational agents that join the network and meet other agents via a random process that is highly influenced by the dynamic network structure and the characteristics of the agents themselves. Thus, networks evolve over time as a stochastic process driven by the individual agents, where the formation of *social ties* among agents are in part endogenously determined, as a function of the current network structure itself, and in part exogenously, as a function of the individual characteristics of the agents. Agents have bounded rationality, i.e. they only have information about other agents they meet over time, they are not able to observe the global network structure or reason about links formed by others, and they are *myopic* in the sense that they take linking decisions without taking possible future meetings into account. We focus on the impact of various exogenous parameters that describe the norms and behaviors of heterogeneous social categories, on the endogenously evolving network structure, and consequently on the emerging social capital. Fig. 1 depicts all such exogenous and endogenous parameters. In the following, we provide definitions for the exogenous parameters under study.

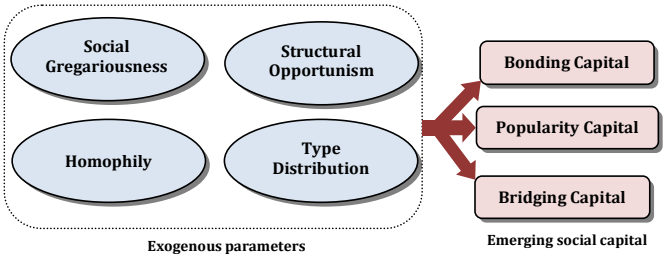


Fig. 1: Framework for the analysis of emerging social capital.

Type Distribution

Agents are heterogeneous as they possess type attributes that designate the social categories to which they belong. A social category is a group of individuals who follow the same norms and behavior; these norms and behaviors are mapped from a high-dimensional latent social space (See Blau's classical book on consolidated and unconsolidated trait dimensions [69]). The experiences of the different interacting social categories in the network are generally not symmetric; thus, social capital is created non-uniformly across them. The type distribution corresponds to the relative population share of different social categories, and represents the fraction of agents of each type in the network. We say that an agent belongs to a *type minority* to qualitatively describe a scenario where the fraction of agents of the corresponding type in the population is small, and we say that an agent belongs to a *type majority* otherwise.

Homophily

Homophily refers to the tendency of agents to connect to other similar-type agents, and it is widely regarded as a pervasive feature of social networks [25], [26], [27]. Various recent empirical studies have shown that homophily stands as an attraction mechanism in forming marriage and cohabitation networks (See Skvoretz work in [70]). We capture the extent to which an agent is *homophilic* by an *exogenous*

homophily index, which we formally define in Section 3. The homophily index can be thought of as the amount of "intolerance" that a certain type of agents have towards making contacts with other types. It can also represent the "closed-mindedness" of a social category; low homophilic tendency means that agents are eager to connect and accept views of other social categories, whereas high homophilic tendency means that agents restrict their social ties to only like-minded individuals.

Social Gregariousness

Some types of agents can be more *sociable* than others, and thus are willing to form more links. Social gregariousness is simply measured by the minimum number of links an agent is willing to make.

Structural Opportunism

Agents in the network are said to be *opportunistic* if they exploit their contacts to find new contacts; thus, agents are more likely to link with the neighbors of their neighbors if they are opportunistic. Structural opportunism can also be interpreted as the *flow of trust* among individuals; each agent trusts the connections of his neighbors more than he trusts others. Opportunism induces *closure* in the network, i.e. connections of an individual are well connected, which on one hand may be thought of as a source of increasing social support for an individual, and on the other hand it can lead to information redundancy, i.e. all connections of an individual possess similar information since they are well connected among each other. Structural opportunism can also correspond to a property of a behavior-dependent meeting mechanism; for instance, users in Twitter are expected to *retweet* the tweets posted by users they follow, which leads to the followers of followers of a certain user to follow him. Similarly, researchers find new papers through the references of papers that they have already cited.

We focus on three different incarnations of social capital that agents gain as the network evolves. These forms of capital differ in terms of the type of advantage they offer to agents, the way they are created and distributed among agents and social categories, and their dependence on the underlying norms and behaviors of social groups, which are abstracted by the exogenous parameters. We focus on *directed networks*, i.e. networks in which ties are formed unilaterally such as Twitter and citation networks. In particular, we focus on the following forms of social capital that emerge in such networks.

Bonding capital

We define the bonding capital as the aggregate informational and social benefits that an individual draws from its direct neighbors in the network. The bonding capital depends only on an individual's ego network (direct connections), and is invariant to the global network structure as long as the local ego network is preserved. The bonding capital increases if the ego network is more homogeneous; individuals are better off when connecting to other similar individuals. This is because more similar individuals are more likely to provide more social support and more relevant information. Since in our model agents form links driven by homophilic incentives, we measure the bonding capital by the agents' utility functions. This form of capital is close to the definitions of Coleman and Bourdieu [1]- [3].

Popularity capital

In our model, we consider a directed social network, thus links are formed by an individual and others also form links towards that individual. Individuals gain bonding capital by forming links to others, and they also gain popularity capital by having other individuals form links to them. The popularity capital represents an individual's ability to influence others. That is, an individual's popularity capital allows it to better spread information and ideas in the network, and also to gain support and agreement on the individual's views and opinions. We measure the popularity capital of an individual by simply counting the number of individuals forming links with that individual.

Bridging capital

Individuals who connect different social categories are able to control the flow of information across those groups and obtain non-redundant information from diverse segregated communities, which allows them to come up with innovations and new ideas [9]. Thus, individuals can acquire a *bridging capital* because of their centrality in the network rather than their popularity or the quality of their ego networks. We measure the bridging capital using a graph theoretic centrality measure, namely, the *betweenness centrality*.

Examples of bonding capital include the knowledge acquired by citing research papers, information and news obtained from following users on Twitter, etc. Popularity capital includes the number of citations associated with a published paper, the impact factor of a journal, the number of followers of a user on Twitter [16], etc. Examples of bridging capital include conducting interdisciplinary research, creating cross-cultural memes on Twitter, etc. Bonding capital helps individuals acquire knowledge, information and support, which allows them to accomplish tasks [15], whereas popularity capital can give financial returns (such as research funds for popular scholars), or intellectual influence (such as in the case of citation networks) [18]. Finally, bridging capital leads to innovation [9], i.e. innovative interdisciplinary research [63]; cross-cultural creative content generated by internet users [66]; or acquisition of non-redundant information about job opportunities in informal organizational networks [8]. Fig. 1 depicts the framework of the paper; we focus on four different exogenous parameters, which abstract the norms and behaviors of social categories, and study their impact on the emergence of the three forms of social capital discussed above.

1.2 Contributions

The central question addressed in this paper is: how do bonding, popularity, and bridging forms of capital emerge simultaneously in an evolving network? We classify our results based on the different forms of capital as follows.

Bonding capital: In Section 4, we study the emergence of bonding capital by characterizing the *ego networks* of individual agents in terms of the time needed for an agent to form its ego network, and the types of agents in that network. We show that majority and opportunistic types are more likely to establish their ego networks in a short time period. Moreover, we show that extreme homophilic tendencies for all social categories is a necessary and sufficient condition for maximizing the aggregate bonding capital of the society – so we show that polarization in a society maximizes bonding capital.

Popularity capital: In Section 5, we show that the acquisition of *popularity capital* displays the *rich-get-richer* effect due to the individuals' structural opportunism. In other words, the popular individuals get more popular as structural opportunism promotes the propagation of trust and reputation across the network, which endows popular agents with "reputational advantages" over time. Furthermore, we show that in tolerant (non-homophilic) societies, an individual's age and the collective gregariousness of social categories are the forces that determine an agent's popularity capital, whereas homophily can create asymmetries in the levels of popularity attained by different social groups.

Bridging capital: In Section 6, we demonstrate (via simulations) the strength of weak ties by showing that when a social category has a different attitude towards homophily compared to all other categories, it ends up being the most central in the network. In particular, we show that the *structural holes* created in extremely homophilic networks represent a potential source of bridging capital for "open-minded" social categories; non-homophilic individuals can fill these holes and broker interactions at the interface between different categories, which allows them to be the most central agents, even if they are neither the most popular nor represent a majority type in the network. Furthermore, we show that in extremely non-homophilic societies, homophilic social categories are the most central; that is, despite the absence of cross-category structural holes, homophilic agents reside in the center of the network, acting as an *information hub* or a *dominant coalition*, through which information diffusion is controlled.

2 Related works

To the best of the authors' knowledge, none of the network formation models in literature have studied the emerging social capital associated with endogenously formed networks. Qualitative studies on social capital by contemporary sociologists such as Coleman, Bourdieu, Lin, Putnam, Portes and Granovetter can be found in [1]- [10], [13]- [16]. These studies give qualitative definitions for the social capital in general (not necessarily networked) societies along with some hypotheses about its emergence in different societies, and they support their hypotheses on the basis of historical and experimental evidence. In addition to the classical work of Coleman, other notable quantitative sociological studies that hinge on graph-theoretic tools to quantify and analyze social capital were conducted by Inkpen and Tsang [71], Burt [72], and Nahapiet and Ghoshal [73]. All those works have developed graph-theoretic notions of social capital (inspired by Freeman's notions of centrality [12]); however, unlike our work, these frameworks are restricted to quantifying the social capital in static networks rather than understanding the evolution and emergence of these quantitative forms of capital.

Empirical studies on the social capital in Online Social Networks (OSN) were carried out in [16], [66] and [68]. These works have given qualitative insights into the emergence of social capital in OSNs mainly based on data, e.g. the number of followers and followees of a user on Twitter, the frequency of interaction and message exchange among users in Facebook, etc. All these works do not come up with mathematical models for the emerging social capital in evolving social networks, thus they neither offer a concrete understanding and explanation for the micro-foundations of social capital, nor offer a counterfactual analysis for different scenarios of network evolution.

While no mathematical model has studied emergent social capital in networks, there exists a voluminous literature focusing on network formation models. Previous works on network formation can be divided into three categories: networks formed based on *random events* [22], [23], [24], [28]- [37], [54], networks formed based on *strategic decisions* [40]- [46], [55], [75], [76] and empirical models distilled by mining networks' data [17]- [19], [21], [47]- [52], [59]. While a fairly large literature has been devoted to developing mathematical models for network formation, a much smaller literature attempts to interpret and understand how networks evolve over time, how individual agents affect the characteristics of such networks, and the "value" of social networking conceptualized in terms of social capital. Probabilistic models based on random events are generative models that are concerned with constructing networks that mimic real-world social networks. In [28]- [39], agents get connected in a pure probabilistic manner in order to realize some degree distribution [28], or according to a *preferential attachment* rule [29] [30]. While such models can capture the basic structural properties of social networks, they fail to explain why and how such properties emerge over time.

In contrast, strategic network formation models such as those in [40]- [44], and our previous works in [45] [46], can offer an explanation for why certain network topologies emerge as an equilibrium of a network formation game. However, these results are limited to studying network *stability* and *efficiency*, and provide only very limited insight into the dynamics and evolution of networks. Moreover, although mining empirical data can help in building algorithms for detecting communities [49]- [52], predicting agents' popularity [48], or identifying agents in a network [47], it is of limited use in understanding how networks form and evolve.

3 Model

3.1 Network model

We construct a model for a growing and evolving social network. Time is discrete. One *agent* (or social actor in the framework of Snijders [54]) enters the network at each moment of time (which is the usual assumption in the graph-theoretic network science literature [29]- [35]); that is, one

paper is published in every time slot in a citation network [57], or one user is creating a Twitter account in every time slot [22]. Agents make link formation decisions that lead to "social graph", based on which we can analyze the "emerging" social capital using the frameworks of Coleman, Bourdieu, Lin, Putnam, Portes and Granovetter [1]- [10].

We index agents by their entry dates $i \in \{1, 2, \ldots, t, \ldots\}$. Agents who have already joined the network by a given date t have the opportunity to form (directed) links; we write G^t for the network that has been formed (by entry and linking) at time t. As we will see, this is a random process $\{G^t\}_{t=0}^{\infty}$. We write $\mathcal{G}^t$ for the space of all possible networks that might emerge at time t and $\Omega_{\mathcal{G}}$ for the space of all possible realizations of the network process. At date $t \in \mathbb{N}$, a snapshot of the network is captured by a *step graph* $G^t = (\mathcal{V}^t, \mathcal{E}^t)$, where $\mathcal{V}^t$ is the set of nodes, $\mathcal{E}^t = \{e_1^t, e_2^t, \ldots, e_{|\mathcal{E}^t|}^t\}$ is the set of edges between different nodes, with each edge e_k^t being an ordered pair of nodes $e_k^t = (i, j)$ $(i \neq j$, and $i, j \in \mathcal{V}^t)$, and $|\mathcal{E}^t|$ is the number of distinct edges in the graph. We emphasize that G^t is a directed graph. Nodes correspond to agents and edges correspond to directed links (or social ties in the terminology of Coleman and Burt [1]- [10]) between the agents. The adjacency matrix of G^t is denoted by $\mathbf{A}_G^t = [A^t(i,j)], A^t(i,j) \in \{0,1\}, A^t(i,i) = 0, \forall i, j \in \mathcal{V}^t$. An entry of the adjacency matrix $A^t(i,j) = 1$ if $(i,j) \in \mathcal{E}_k^t$, and $A^t(i,j) = 0$ otherwise. If $A^t(i,j) = 1$, then agent i has formed a link with agent j, and we say that j is a "followee" of i, and i is a "follower" of j. The directed nature of a link indicates the agent forming the link, and only this agent obtains the *social benefit* of linking and pays the link cost. The *indegree* of agent i is the number of links that are initiated towards i, denoted by $\deg_i^-(t)$, while the *outdegree*, denoted by $\deg_i^+(t)$, is the number of links initiated by agent i. Agents i and j are connected if there is a path of edges from i to j (ignoring directions); a *component* is a maximal connected set of agents. A *singleton component* is a component comprising one agent. The number of non-singleton components of a step graph G^t is denoted by $\omega(G^t)$, where $1 \leq \omega(G^t) \leq |\mathcal{V}^t|$.

Each agent i is described by a type attribute θ_i, which belongs to a finite set of types $\theta_i \in \Theta, \Theta = \{1, 2, 3, \ldots, |\Theta|\}$, where $|\Theta|$ is the number of types. The type of an agent abstracts the social category to which it belongs; and all agents belonging to the same social category follow the same behavior which are mapped from a high-dimensional latent social space [69]. The set of type-k agents at time t is denoted by $\mathcal{V}_k^t$, where $\mathcal{V}^t = \bigcup_{k=1}^{|\Theta|} \mathcal{V}_k^t$, and $\mathcal{V}_k^t \bigcap \mathcal{V}_m^t = \emptyset, \forall k, m \in \Theta, k \neq m$. We define the length-$L$ *ego network* of agent i at time t, $G_{i,L}^t$, as the subgraph of G^t induced by node i, and any node j that can be reached via a directed path of length less than or equal to L starting from node i. In this paper, an "ego network" generally refers to the length-1 ego network of an agent.

There are three aspects of network formation: agents enter; agents meet; agents form links. Entry is governed by a stationary random process; meeting is governed by a non-stationary random process; linking is governed by active choices. We describe each of these processes in the following subsections.

3.2 The Entry Process

At time 0 the network is empty ($G^0 = \emptyset$). Agents enter one at a time at each date t according to a stationary stochastic process $\lambda(t) = \{\theta_t\}_{t \in \mathbb{N}}$, with a sample space $\Lambda = \Theta^{\mathbb{N}}$, i.e. $\Lambda = \{(\theta_1, \theta_2, \ldots) : \theta_t \in \Theta, \forall t \in \mathbb{N}\}$. We assume that the types of agents are independent and identically distributed (θ_i and θ_j are independent for all $i \neq j$), and that the agents' *type distribution* is $\mathbb{P}(\theta_i = k) = p_k$, where $\sum_{k \in \Theta} p_k = 1$, so, $\lambda(t)$ is a *Bernoulli scheme*. At date t, the expected number of type-k agents in the network is $p_k t$, the total number of agents is t, i.e. $|\mathcal{V}^t| = t$, and $\lim_{t \to \infty} \frac{|\mathcal{V}_k^t|}{|\mathcal{V}^t|} = p_k$. Using *Borel's law of large numbers*, we know that

$$\mathbb{P}\left(\lim_{t \to \infty} \frac{1}{t} \left|\mathcal{V}_k^t\right| = p_k\right) = 1.$$

In other words, for a sufficiently large network size (and age t), the actual fraction of agents of each type in the network converges almost surely to the prior type distribution of the Bernoulli scheme.

3.3 The Meeting Process

At each moment in time t, every agent i who is alive at time t (i.e. $i \leq t$) meets one other agent $m_i(t)$ (identified by its entry date). The meeting process is random (described in detail below); we write $M_i(t) = \{m_i(t)\}_{t=i}^{i+T_i-1}$ for the *meeting process* of agent i. The meeting process may stop at some finite time T_i (the stopping time) or continue indefinitely (in which case $T_i = \infty$). The sample space of the meeting process is given by $\mathcal{M}$. Agents meet other agents who belong to one of two *choice sets* (this terminology was first introduced by Bruch and Mare in [53]), namely the set of *followees of followees* and the set of *strangers*. Unlike the entry process, which is stationary, the meeting process depends on the current network, which in turn depends on the past history: the probability that agent i meets agent j at time t depends on their relative positions in the network at time t, which in turns depend on the sequence of meetings for both agents up to time $t-1$. Moreover, the probability that a certain sample path of the meeting process occurs depends on all the exogenous parameters shown in Fig. 1.

Given a time t, an agent i alive at time t, and the existing network G^t, write $\mathcal{N}_{i,t}^+$ for the set of followees of i and $\mathcal{K}_{i,t} = \left(\bigcup_{j \in \mathcal{N}_{i,t-1}^+} \mathcal{N}_{j,t-1}^+ / \{i\}\right) / \mathcal{N}_{i,t-1}^+$ for the set of *followees of followees* of agent i. Everyone who is neither a followee nor a followee of a followee is a *stranger*. (Note that the newly entering agent t is always a stranger.) At time t agent i meets either a followee of a followee or a stranger; the probability of meeting a followee of a followee (if one exists) is an exogenous parameter $\gamma_k \in [0,1]$ (where k is the type of i), which we think of as structural opportunism (taking advantage of opportunities [1]), where $\gamma_k = 1$ for fully opportunistic agents, and $\gamma_k = 0$ for fully non-opportunistic agents.

Denote the set of type-k followees of agent $i \in \mathcal{V}^t$ by $\mathcal{N}_{i,t}^{+,k}$, and the set of all followees of i as $\mathcal{N}_{i,t}^+ = \bigcup_{k=1}^{|\Theta|} \mathcal{N}_{i,t}^{+,k}$,

1. The parameter γ_k can also be thought of as a realization of the *triadic closure*; the flow of "trust" among connected individuals [54], or as an exploration-exploitation behavior; an agent either explores the network or exploits his current connections with different probabilities.

where $|\mathcal{N}_{i,t}^{+}| = \deg_i^{+}(t)$. Similarly, we denote the followers of agent i by $\mathcal{N}_{i,t}^{-}$, where $|\mathcal{N}_{i,t}^{-}| = \deg_i^{-}(t)$. Define the set $\mathcal{K}_{i,t} = \left(\bigcup_{j\in\mathcal{N}_{i,t-1}^{+}} \mathcal{N}_{j,t-1}^{+}/\{i\}\right)/\mathcal{N}_{i,t-1}^{+}$ as the set of *followees of followees* of agent i at time t, and the set $\bar{\mathcal{K}}_{i,t} = \mathcal{V}^t/\left\{\mathcal{K}_{i,t}\bigcup\mathcal{N}_{i,t-1}^{+}\bigcup i\right\}$ as the set of *strangers* to agent i at time t. The set of same type followees of followees is denoted as $\mathcal{K}_{i,t}^{\theta_i}$. Let $N_i^s(t) = |\mathcal{N}_{i,t}^{+,\theta_i}|$, $N_i^d(t) = \deg_i^{+}(t) - N_i^s(t)$, $K_i(t) = |\mathcal{K}_{i,t}|$, $K_i^s(t) = |\mathcal{K}_{i,t}^{\theta_i}|$, and $K_i^d(t) = K_i(t) - K_i^s(t)$.

For $t \geq i$, if there are no followees of followees, then i meets a stranger with uniform probability. If there are followees of followees, then i meets a followee of a followee with probability γ_k (and uniform over this choice set) and meets an agent picked uniformly at random from the network with probability $1-\gamma_k$, i.e.

$$\mathbb{P}\left(m_i(t) \in \mathcal{K}_{i,t} \,|\mathcal{K}_{i,t} \neq \emptyset\right) = \gamma_{\theta_i} + (1-\gamma_{\theta_i})\frac{K_i(t)}{t-1},$$

and

$$\mathbb{P}\left(m_i(t) \in \bar{\mathcal{K}}_{i,t} \,|\mathcal{K}_{i,t} \neq \emptyset\right) = (1-\gamma_{\theta_i})\frac{t-1-K_i(t)}{t-1},$$

whereas $\mathbb{P}\left(m_i(t) \in \bar{\mathcal{K}}_{i,t} \,|\mathcal{K}_{i,t} = \emptyset\right) = 1$. Note that since a new agent enters at each time step, and such an agent is a stranger to all other agents, then we have $\mathbb{P}\left(\bar{\mathcal{K}}_{i,t} \neq \emptyset\right) = 1$ for any time step t. At each time t, i meets a new agent; i may or may not form a link to this agent. In addition some agents may meet agent i, but i does not form links to those agents. The meeting process realizes the *limited-observability* of agents over time, i.e. agent i reasons about forming social ties with only the agent it meets at time t, and cannot observe the global network structure or the types of all agents it does not meet. This is different from the complete information and complete observability network formation games in [29], or the preferential attachment models in [40] which assumes that the linking behavior of a newly entering agent relies on its knowledge of all the degrees of other agents.

3.4 The Linking Process

When agent i meets agent $m_i(t)$ at time t, it observes the type of $m_i(t)$ and decides whether or not to form a link with $m_i(t)$ (Thus true types of agents who meet are revealed). Agents draw benefits by linking to others but link formation is costly. Agents optimize so they form a new link if the marginal benefit of that link exceeds marginal cost. The marginal benefit depends on existing links and on types; we assume that linking to agents of the same type is (weakly) better than linking to agents of a different type – this is homophily. For simplicity we assume marginal cost of linking is a constant c.

We assume *local externalities*, i.e. linking benefits do not flow to indirect contacts, so i derives benefits only from its (direct) neighbors. For simplicity we assume that the utility depends only on the number of followees of the same type $N_i^s(t)$ and the number of followees of different types $N_i^d(t)$, and has the form

$$u_i^t\left(G_{i,1}^t\right) = v_{\theta_i}\left(\alpha_{\theta_i}^s N_i^s(t) + \alpha_{\theta_i}^d N_i^d(t)\right) - c\sum_{j=i}^{t-1} a_i^j, \quad (1)$$

where $a_i^t \in \{0,1\}$ is the action of agent i at time t; $a_i^t = 1$ means that i links to $m_i(t)$, and $a_i^t = 0$ means that i decides not to link to $m_i(t)$, and $\sum_{j=i}^{t-1} a_i^j = \left(N_i^s(t) + N_i^d(t)\right)$, $\alpha_{\theta_i}^s \geq \alpha_{\theta_i}^d, \forall\theta_i \in \Theta$ are the (type-specific) linking benefits, $v_{\theta_i}(x) : x \to \mathbb{R}^+$ is the (type-specific) *social benefit aggregation* function. For convenience, we assume that $v_{\theta_i}(x)$ is strictly concave, twice continuously differentiable, monotonically increasing in x, and $v_{\theta_i}(0) = 0$. That is, the marginal benefit of forming links diminishes as the number of links increases. This corresponds to the fact that agents do not form an infinite number of links in the network, but rather form a "satisfactory" number of links. As shown in (2), i decides to link to $m_i(t)$ only if the marginal utility is positive. Note that i's link formation decisions depend not only on the types of agents it meets, but also on the order with which it meets these agents.

Agent i will form a link to $m_i(t)$ exactly when doing so creates a network that yields higher utility for him. Agents are *myopic* and form links without taking the future into account. This seems to us to be a realistic description of behavior in social networks.

3.5 Homophily and Social Gregariousness

We characterize the extent of homophily and gregariousness of different types of agents using exogenous (ex-ante) quantities. To quantify the gregariousness of type-θ agents, we define the parameters $L_\theta^*(\alpha)$ and $\bar{L}_\theta^*(\alpha)$ for $\alpha \in \mathbb{R}_+ \cup \{0\}$ as

$$L_\theta^*(\alpha) = \arg\max_{x\in\mathbb{Z}} v_\theta(x\alpha_\theta^s + \alpha) - xc, \quad (3)$$

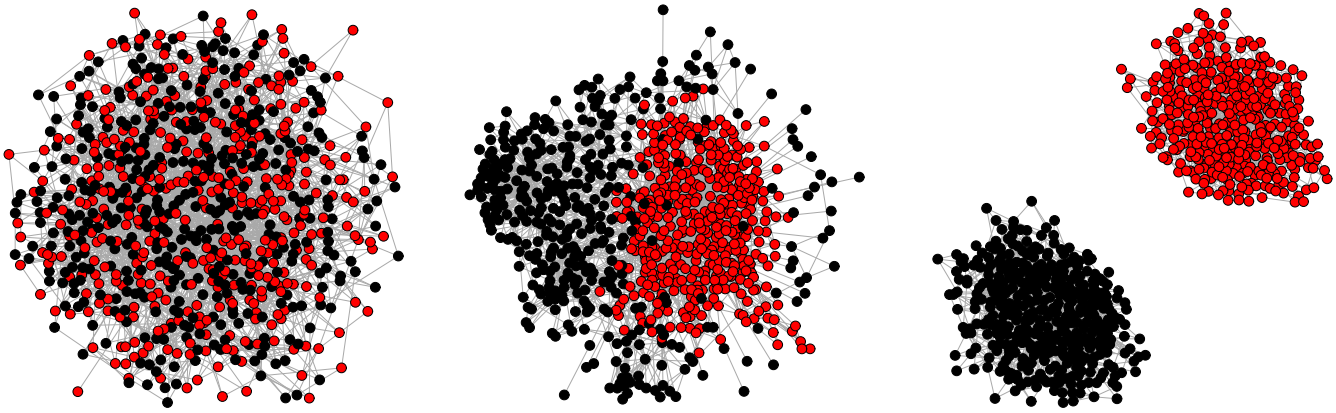
and

$$\bar{L}_\theta^*(\alpha) = \arg\max_{x\in\mathbb{Z}} v_\theta(x\alpha_\theta^d + \alpha) - xc. \quad (4)$$

That is, $L_\theta^*(\alpha)$ is the number of similar-type links that a type-θ agent with an aggregate social benefit of α needs to form in order to saturate its utility function (i.e. reach a point in which adding more links provides no marginal benefit), and $\bar{L}_\theta^*(\alpha)$ is defined similarly but with respect to different-type links. It follows from the concavity of $v_\theta(.)$ that both $L_\theta^*(\alpha)$ and $\bar{L}_\theta^*(\alpha)$ are bounded from above for all $\alpha \in \mathbb{R}$. The parameter $L_\theta^*(0)$ can be interpreted as the "*minimum number of links that a type-θ agent desires to create upon entering the network*". Hence, we use the parameter $L_\theta^*(0)$ to capture the social gregariousness of type-θ agents.

Our conception of homophily is based on the framework proposed by Coleman [37]. In particular, we define the *exogenous homophily index* of type-θ agents as the "*minimum fraction of similar-type links that a type-θ agent desires to create in its followee set upon entering the network*". In the light of this definition, the exogenous homophily index can be quantified straightforwardly using the gregariousness parameters in (3) and (4). That is, since $\bar{L}_\theta^*(\alpha)$ is monotonically decreasing in $\alpha \in \mathbb{R}_+ \cup \{0\}$, then the maximum number of different-type links that a type-θ agent can form is $\bar{L}_\theta^*(0)$. An agent with $\bar{L}_\theta^*(0)$ different-type links attains a social benefit of $\alpha_\theta^d \bar{L}_\theta^*(0)$, and hence its utility function is saturated by forming links with $L_\theta^*(\alpha_\theta^d \bar{L}_\theta^*(0))$ similar-type agents. Therefore, the exogenous homophily index is given by

$$h_\theta = \frac{L_\theta^*(\alpha_\theta^d \bar{L}_\theta^*(0))}{L_\theta^*(\alpha_\theta^d \bar{L}_\theta^*(0)) + \bar{L}_\theta^*(0)}, \quad (5)$$



(a) $h_1 = h_2 = 0$. (b) $h_1 = 1, h_2 = \frac{2}{3}$. (c) $h_1 = h_2 = 1$.

Fig. 2: Snapshots for a social network generated via our model at t = 900 with $L_1^*(0) = L_2^*(0) = 3$, $p_1 = p_2 = \frac{1}{2}$ and $|\Theta| = 2$ for various values of the exogenous homophily indices. The types of agents are distinguished by colors. In (a) there is lots of cross-linking, in (b) the network is very dispersed, in (c) the network is separated into two homogeneous subnetworks.

$$a_i^t = \mathbf{1}_{\left\{\Delta u_i^t(m_i^t; N_i^s(t), N_i^d(t)) > 0\right\}},$$

$$\Delta u_i^t(m_i^t; N_i^s(t), N_i^d(t)) = v_{\theta_i}\left(\alpha_{\theta_i}^s\left(N_i^s(t) + \mathbf{1}_{\left\{\theta_{m_i(t)} = \theta_i\right\}}\right) + \alpha_{\theta_i}^d\left(N_i^d(t) + \mathbf{1}_{\left\{\theta_{m_i(t)} \neq \theta_i\right\}}\right)\right) - v_{\theta_i}\left(\alpha_{\theta_i}^s N_i^s(t) + \alpha_{\theta_i}^d N_i^d(t)\right) - c. \quad (2)$$

where $0 \leq h_\theta \leq 1$. The exogenous homophily index h_θ reflects the minimum fraction of similar-type agents that a type-θ agent can have in its followee set while having its utility function saturated. When type-θ agents are indifferent to the types of agents they connect to, i.e. type-θ agents are extremely non-homophilic, then we have that $L_\theta^*(\alpha) = \bar{L}_\theta^*(\alpha)$, $L_\theta^*(\alpha_\theta^d \bar{L}_\theta^*(0)) = 0$, and $h_\theta = 0$, which means that agent i can get satisfied by connecting to a set of followees that does not contain any similar-type followee. On the other hand, if type-θ agents restrict their links to similar-type agents only (type-θ agents are extremely homophilic), then we have $\bar{L}_\theta^*(\alpha) = 0$, $L_\theta^*(\alpha_\theta^d \bar{L}_\theta^*(0)) = L_\theta^*(0)$, and $h_\theta = 1$. As shown in Fig. 2, the more homophilic the agents are, the more segregated the network will be in the sense that different types of agents will be weakly connected.

3.6 Summary: Exogenous Parameters

In summary, our model involves four exogenous parameters:

- *Homophily*: the homophily of type-k agents is captured by the exogenous homophily index h_k.
- *Social gregariousness*: the gregariousness of type-k agents is captured by $L_k^*(0)$.
- *Structural opportunism*: the parameter γ_k reflects the extent of structural opportunism for type-k agents.
- *Type distribution*: the fraction of type-k agents in a large network (relative population share) is given by p_k.

Throughout this paper, we will use the notion of *first-order stochastic dominance* (FOSD). We say that a pdf (or pmf) $f(x)$ first-order stochastically dominates a pdf $g(x)$ if and only if $G(x) \geq F(x), \forall x$, with strict inequality for some values of x, where $F(x)$ and $G(x)$ are the cumulative density functions. We write $X \succeq Y$ for the two random variables X and Y when X first-order stochastically dominates Y.

4 Bonding Capital

In this section we focus on bonding capital; we discuss popularity capital and bridging capital in following sections.

4.1 Ego network formation time

Unlike previous works where link formation is a one-shot process (which is the case in [24], [30], [34]- [41], [43], and [44]), links (and consequently the bonding capital) are created over time in our model; individuals meet others and decide to establish connections until they forms a "satisfactory" ego network/network of followees. Hence, individuals build up their bonding capital gradually over time, and the time needed to reach a steady-state utility (form an ego network) is an essential component for characterizing the emergence of the bonding capital, and would be a very crucial if agent's exit from the network is considered. In this section, we characterize the bonding capital in terms of the time needed for the emergence of an ego network, as well as the utility resulting from bonding to that ego network.

Based on the definition of the utility function in (1) and (2), we know that there exists a finite number of connections after which an agent stops forming links. The time horizon over which the agent forms its ego network is random and depends on all the exogenous parameters. For an agent i, the ego network formation time (EFT) T_i is a random function of the exogenous parameters, defined as

$$T_i \triangleq$$

$$\inf\left\{t \in \mathbb{N} : u_i^{\tau}\left(G_{i,1}^{t}\right) \geq u_i^{\tau}\left(G_{i,1}^{t} \cup j\right), \forall \theta_j \in \Theta, \tau > t\right\} - i + 1. \quad (6)$$

We emphasize that T_i is random: it depends on the network formation process. We characterize the time spent by an agent in the process of forming his ego network/network of followees in terms of the probability mass function (pmf) of T_i. [2] We denote the pmf of T_i as $f_{T_i}(T_i) : \mathbb{N} \to [0,1]$. The expected ego network Formation Time (EEFT) $\overline{T}_i$ conditioned on agent i's type is given by

$$\overline{T}_i = \mathbb{E}_{\Omega_{\mathcal{G}}}\left[T_i \,|\theta_i\right], \quad (7)$$

where $\mathbb{E}_{\Omega_{\mathcal{G}}}[.]$ is the expectation operator, and the expectation is taken over all realizations of the graph process (we drop the subscript $\Omega_{\mathcal{G}}$ in the rest of our analysis). We say that agent i is *socially unsatisfied* if $T_i = \infty$; a socially unsatisfied agent is an agent that never satisfies its gregariousness requirements, i.e. agent i is socially unsatisfied if $\deg_i^{+}(t) < L_{\theta_i}^{*}(0), \forall t \geq i$. Such an agent keeps searching for followees forever. In the following Lemma, we specify the necessary and sufficient conditions under which a newly entering agent has a positive probability of becoming socially unsatisfied. The proofs for all Theorems in this paper can be found in the online appendix in [74].

Lemma 1. In order that agent i becomes socially unsatisfied with positive probability, it is necessary and sufficient that $\gamma_{\theta_i} = 1$ and $0 < h_{\theta_i} < 1$. □

This Lemma says that an agent gets unsatisfied if and only if it is not extremely homophilic and at the same time does not explore the strangers' choice set in its meeting process. In such a scenario, an agent's meeting process is governed by the actions taken previously by his neighborhood, which may not allow that agent to meet with other agents of diverse types. Unless otherwise stated, we assume that $\gamma_k < 1, \forall k \in \Theta$, thus agents never get trapped and all agents have a finite EFT. In the rest of this subsection, we characterize the EFT. We start by characterizing the EFT for extreme cases of agents' homophily in the following Theorem.

Theorem 1.

1) If $h_k = 0, \forall k \in \Theta$, then the EFT for agent i is equal to $T_i = L_{\theta_i}^{*}(0)$ almost surely.
2) If $h_k = 1, \forall k \in \Theta$, then the distribution of the EFT for every agent i conditioned on its type converges to a *steady-state distribution*, i.e. $\lim_{i\to\infty} f_{T_i}\left(T_i \,|\theta_i = k\right) \to f_T^k(T)$, and the EEFT for an agent i conditioned on its type $\overline{T}_i = \mathbb{E}[T_i \,|\, \theta_i = k]$ converges as follows

$$\lim_{i\to\infty} \overline{T}_i = \frac{1}{p_k} + \frac{L_k^{*}(\alpha_k^{s})}{(1-\gamma_k)\, p_k + \gamma_k}. \quad \square$$

Thus, the EFT for agents joining a large network only depends on their types. Theorem 1 says that when the agents are *not* homophilic, there is no uncertainty in the ego network formation process, then both the number of links and the EFT are equal to $L_{\theta_i}^{*}(0)$ almost surely. This "deterministic" EFT is independent of the network, and only depends on the agent's gregariousness. That is, if $h_k = 0, \forall k \in \Theta$, then an agent's journey in the network is determined by how it values linking, and not by the network structure or the actions of others. If agents are more *sociable*, i.e. are more gregarious, then they will spend more time searching for followees, yet this time is deterministic and only depends on parameters that are determined by the agent and not the network. On the other hand, if agents are extremely homophilic, then the agent's journey in the network will depend randomly on meetings with other agents with whom they do not form any links. It is clear from Theorem 1 that the EEFT of extremely homophilic agents depends on the type distribution and opportunism, in addition to gregariousness. We emphasize these dependencies in the following corollary.

Corollary 1. *(Gregarious agents and minorities search for followees longer, opportunistic agents search shorter)* If $h_k = 1, \forall k \in \Theta$, $\tilde{L}_{\theta_i}^{*}(0) \geq L_{\theta_i}^{*}(0)$, $\tilde{p}_{\theta_i} \geq p_{\theta_i}$, and $\tilde{\gamma}_{\theta_i} \geq \gamma_{\theta_i}$, then for an agent i entering an asymptotically large network we have that

$$T_i\left(p_{\theta_i}, \gamma_{\theta_i}, L_{\theta_i}^{*}(0)\right) \preceq T_i\left(p_{\theta_i}, \gamma_{\theta_i}, \tilde{L}_{\theta_i}^{*}(0)\right),$$

$$T_i\left(p_{\theta_i}, \gamma_{\theta_i}, L_{\theta_i}^{*}(0)\right) \succeq T_i\left(\tilde{p}_{\theta_i}, \gamma_{\theta_i}, L_{\theta_i}^{*}(0)\right),$$

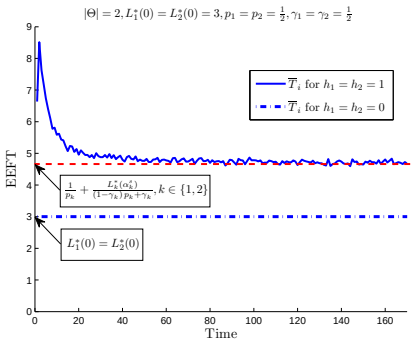
$$T_i\left(p_{\theta_i}, \gamma_{\theta_i}, L_{\theta_i}^{*}(0)\right) \succeq T_i\left(p_{\theta_i}, \tilde{\gamma}_{\theta_i}, L_{\theta_i}^{*}(0)\right),$$

where $T_i\left(p_{\theta_i}, \gamma_{\theta_i}, L_{\theta_i}^{*}(0)\right)$ is the EFT associated with the exogenous parameter tuple $\left(p_{\theta_i}, \gamma_{\theta_i}, L_{\theta_i}^{*}(0)\right)$. □

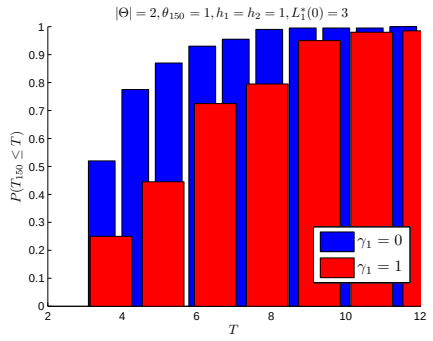
Note that stochastic dominance implies domination in mean. That is, if $T_i\left(p_{\theta_i}, \gamma_{\theta_i}, L_{\theta_i}^{*}(0)\right) \preceq T_i\left(p_{\theta_i}, \gamma_{\theta_i}, \tilde{L}_{\theta_i}^{*}(0)\right)$, then $\overline{T}_i\left(p_{\theta_i}, \gamma_{\theta_i}, L_{\theta_i}^{*}(0)\right) \leq \overline{T}_i\left(p_{\theta_i}, \gamma_{\theta_i}, \tilde{L}_{\theta_i}^{*}(0)\right)$. Moreover, stochastic dominance implies domination of the expectation of any increasing function of the EFT; if the bonding capital is a decreasing function of the EFT, then one can infer the impact of the exogenous parameters on the expected bonding capital directly from the results of Corollary 1.

Corollary 1 says that in homophilic societies, the EFT of a social category *increases (in the sense of FOSD)* as the gregariousness of that group increases. This is intuitive since the more followees an agent is willing to follow, the longer it takes to find those followees. Moreover, agents belonging to minorities are expected to spend more time in the link formation process. This is again intuitive since when the fraction of similar-type agents in the population is small, each agent would need to meet a longer sequence of agents in order to find similar-type followees. Finally, the EFT decreases in the sense of FOSD as structural opportunism increases. This is because once the agent becomes attached to a network component of similar-type agents, it is then better to be opportunistic and keep meeting the followees of followees who are guaranteed to be similar-type agents, rather than meeting strangers with uncertain types. In this context, structural opportunism captures what Mayhew calls "structuralist" homophily effects in [26], and what Kossinets and Watts refer to as "induced homophily" in [27], which corresponds to the fact that similar-type agents are more likely to "meet" when agents are opportunistic.

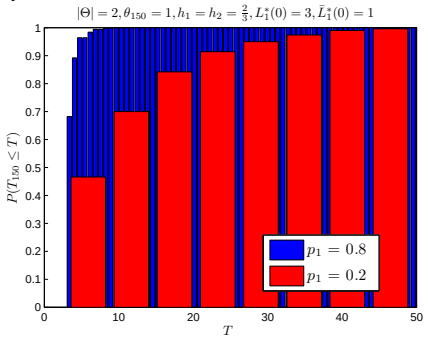
2. Note that T_i can be thought of as the *stopping time* of the linking process. This can be easily proven by showing that the event $T_i = T$ only depends on the history of meetings and link formation decisions up to time T.



(a) Impact of homophily on the EEFT.



(b) The EFT cdf for $\gamma_1 = 0$ and $\gamma_1 = 1$.



(c) The EFT cdf for $p_1 = 0.2$ and $p_1 = 0.8$.

Fig. 3: Stochastic ordering of the EFT with respect to the exogenous parameters.

In the following Corollary, we show that the meeting process, encoded in the structural opportunism, plays a more crucial role for "minor" types.

Corollary 2. If $h_k = 1, \forall k \in \Theta$, then for an agent i entering an asymptotically large network, the following is satisfied:

1) If agent i belongs to a minor type ($p_{\theta_i} \to 0$), then we have that $\lim_{\gamma_{\theta_i} \to 1} \overline{T}_i = \frac{1}{p_{\theta_i}} + L_{\theta_i}^*(0)$, and $\lim_{\gamma_{\theta_i} \to 0} \overline{T}_i = \frac{L_{\theta_i}^*(0)}{p_{\theta_i}}$.
2) If agent i belongs to a major type ($p_{\theta_i} \to 1$), then for every γ_{θ_i} we have that $\lim_{\gamma_{\theta_i} \to 0} \overline{T}_i = L_{\theta_i}^*(0)$. □

Thus, if minor types *exploit* their current connections to form new links, their EEFT becomes inversely proportional to their population size p_{θ_i} with an *additive* gregariousness parameter, whereas if the minor types *explore* the network by meeting strangers, their EEFT becomes inversely proportional to their population size p_{θ_i} with a *multiplicative* gregariousness parameter. Therefore, minor types need to be more opportunistic for their EEFT to decrease. On the other hand, agents belonging to a "major" type with $p_{\theta_i} \to 1$ have an EEFT $\overline{T}_i \to L_{\theta_i}^*(0)$ regardless of their level of opportunism. Thus, the EFT of major types is less affected by the meeting process.

Fig. 3 reports simulations that illustrate the results of Theorem 1 and Corollary 1. In Fig. 3(a), we can see that the EEFT in an extremely homophilic society is greater than that of a non-homophilic society, and as the network grows, the EEFT for homophilic agents converges to the value specified by Theorem 1. In Fig. 3(b), we plot the cdf of the EFT for homophilic agents with different levels of opportunism, and we can see that the EFT of non opportunistic agents stochastically dominates that of opportunistic agents. Similarly, we demonstrate the impact of the type distribution in Fig. 3(c).

4.2 Ego network characterization: homophily and structural holes

In the previous subsection we have characterized the time needed for individuals to form their local ego networks, and thus realize a bonding capital. A common aspect in the definitions of bonding capital by Putnam [6], Bourdieu [3], Coleman [1], Fischer [13], and Cobb [14], is that it corresponds to the *social support* that individuals obtain through networking. Social support includes companionship, information exchange, emotional and instrumental support. In our model, agents derive social support from their followees; and such support is larger when the agent and its followees are of the same type, i.e. if an agent connects with same-type agents, they will acquire more relevant information [15]. Thus, the types of agents in an agent's ego network determine its bonding capital. Based on this, we consider an agent's utility function, which represents the agent's net aggregate linking benefit, as an operational measure for the bonding capital accumulated by that agent. The bonding capital accumulated by an agent i at time t is simply measured by its utility u_i^t, whereas the bonding capital of type-k agent is measured by their average utility $U_k^t = \frac{1}{|\mathcal{V}_k^t|} \sum_{j \in \mathcal{V}_k^t} u_j^t$, and the bonding capital of all agents in the network is $U^t = \frac{1}{t} \sum_{i \in \mathcal{V}^t} u_i^t$.

We note that a larger ego network does not imply greater social or informational support. In fact, an agent might establish an ego network that comprises many different-type agents and will then have to pay the cost (time, effort,

etc) to maintain the links with them while getting little social/informational support. For instance, a Twitter user who follows many accounts spreading information that is not relevant to the user's interests leads to low bonding capital: the user then spends time following such accounts but gets low informational benefits. The utility of each agent in a steady-state ego network is a measure for the support that an individual can obtain from other individuals in his local personal network. In the following Theorem, we show that maximum bonding capital is only achieved in societies with extreme homophily.

Theorem 2. (Homophily induces structural holes) Assume $h_l > 0, \forall \ell \in \Theta$. In order that the total average utility U^t converges to the optimal value $\overline{U}^*$ as the network grows without bound it is necessary and sufficient that $h_l = 1, \forall \ell \in \Theta$. If this is the case then the network at any time step will be disconnected almost surely and have at least $|\Theta|$ non-singleton components. □

If all agents are extremely homophilic, then a disconnected network that maximizes the achieved utility always emerges, and such a network is always disconnected even with the limited observability of the meeting process. A disconnected network obviously entails *structural holes* as defined by Burt [8] [9]: same-type agents form connected components that do not communicate with other types of agents, thus different types of agents do not exchange ideas and information. As shown in Fig. 4, the optimal total average utility is only achieved when agents are extremely intolerant towards different-type agents. We can also see that both the total average utility, and the ELFT (reflected in the time needed to reach a steady-state average utility in Fig. 4) of a social category exhibit a non-monotonic behavior with respect to the homophily index.

Thus, maximizing the bonding capital in homophilic societies implies the presence of structural holes. For any network with non-extremely homophilic agents, the limited observability of agents dictated by the meeting process allows the agents to fill the network's structural holes. In other words, what makes the network connected is that not all similar-type agents observe each other at each time step, but they can potentially meet different-type agents with which they decide to connect. If the meeting process allows unlimited observability, i.e. $m_i(t) = \mathcal{V}^t/\{i\}$, then the agents will always converge to a disconnected network with $|\Theta|$ non-singleton components.

The major conclusion drawn from this section is that homophily leads agents to reside in more homogeneous ego networks, but also leads the agents to wait longer in order to establish their ego networks, and induces structural holes in the global network structure. Thus, on one hand homophily unifies the local structure of the network by gathering people with similar traits together, but on the other hand it divides the global network structure since dissimilar social categories become weakly connected. This creates another potential source of capital, namely a *bridging capital*, which we discuss in Section 6.

5 Popularity Capital

Since in our model we consider directed networks, we distinguish between conventional bonding capital, which is

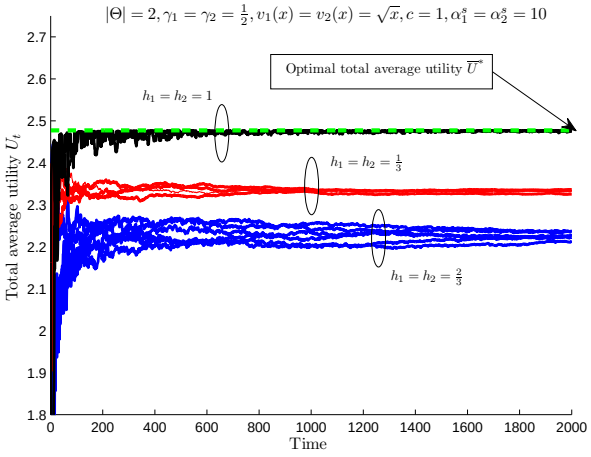


Fig. 4: The total average utility for various realizations of the evolving social network.

realized by homogenous networks of like-minded people that provides social support for the individual, and *popularity capital*, which corresponds to the individual's influence in the network that is gained by supporting others. Popularity is an important form of social capital that represents an individual's influence on a social category; an individual's ability to spread opinions, information, and ideas; and also an individual's acquisition for group support. For instance, users of Twitter acquire popularity measured by the number of followers, which allows them to express opinions, problems, and experiences, and acquire emotional support provided by their online support groups (followers). Followers retweet the tweets posted by users, which allows those users to spread their ideas and opinions [16]. Similarly, the popularity of researchers measured by the number of citations or the h-index allows those researchers to promote for new research ideas and directions. In this section we study popularity capital and connect it to preferential attachment, which is a central concept in network science.

The popularity of agent i at time t is simply given by $\deg_i^-(t)$. We say that the *popularity growth rate* of agent i is $O\left(g(t)\right)$ if $\lim_{t\to\infty} \frac{\mathbb{E}[\deg_i^-(t)]}{g(t)} = 1$, where the expectation is taken over all realizations of the graph process given that agent i enters with a type θ_i. (Note that the growth rate is only uniquely defined "near infinity".) The *popularity distribution* (sometimes called the degree distribution [24] [33] [36]) is denoted by $f_d^t(d)$, and corresponds to the fraction of agents with a popularity level of d at time t, i.e. $f_d^t(d) = \frac{1}{t}\left|\{i\,|\deg_i^-(t) = d, i \in \mathcal{V}^t\}\right|$. For a given type k, $f_d^{t,k}(d)$ denotes the popularity distribution of type-k agents at time t: $f_d^{t,k}(d) = \frac{1}{|\mathcal{V}_k^t|}\left|\{i\,|\deg_i^-(t) = d, i \in \mathcal{V}_k^t\}\right|$. Let $\Delta\deg_i^-(t)$ be the number of followers gained by agent i at time t, i.e. $\Delta\deg_i^-(t) = \deg_i^-(t) - \deg_i^-(t-1)$.

Preferential attachment has been used to explain the underlying mechanism of networks growth [24], [29]- [31], [34]- [36]. All of these previous papers model agents as forming links only once; in our model, agents may form links many times. More importantly, all of these previous

models *impose* preferential attachment as a behavioral rule (so network growth is viewed as a conventional *stochastic urn process*); in our model, preferential attachment *emerges endogenously*.

To fix ideas, we first provide a general definition of preferential attachment that will be adopted in what follows.

Definition 1. (*Preferential attachment*) We say that preferential attachment emerges in the network formation process if $\deg_i^-(t) \geq \deg_j^-(t)$ implies $\Delta\deg_i^-(t) \succeq \Delta\deg_j^-(t)$. □

In words: preferential attachment means that agents who are more popular at a given time are likely to become even more popular in the future.

5.1 Popularity capital in tolerant societies

We begin by studying popularity capital in societies with extreme exogenous homophily index for all types of agents given by $h_k = 0, \forall k \in \Theta$. It seems natural to refer to such societies as tolerant (rather than totally non-homophilic). We study the factors that create inequality of popularity capital in tolerant societies. In the following Theorem, we begin by studying the impact of the exogenous network parameters on the popularity growth rates.

Theorem 3. (*Popularity growth in tolerant societies*) For a tolerant society popularity growth rates enjoy the following properties:

- For $\gamma_k = 0, \forall k \in \Theta$, the popularity of any agent i grows logarithmically with time, i.e. $\mathbb{E}[\deg_i^-(t)]$ is $O\left(\bar{L}\log(t)\right)$, where $\bar{L} = \sum_{m\in\Theta} p_m L_m^*(0)$.
- For $\gamma_k = 1, \forall k \in \Theta$, the popularity of any agent i grows at least sublinearly with time, i.e. $\mathbb{E}[\deg_i^-(t)]$ is at least as fast as $O\left(t^b\right)$, where b is given in Appendix E in [74] and is the same for all types of agents. □

This Theorem demonstrates the impact of opportunism and gregariousness on popularity accumulation. On one hand, the popularity of agents in non-opportunistic societies grows logarithmically with time – very slowly. On the other hand, the popularity of agents in opportunistic societies grows sublinearly with time – again slowly, but much faster than for non-opporunistic agents. Thus, opportunism has an enormous influence on popularity. As we show below, this is a consequence of preferential attachment.

Corollary 3. *(Emergence of preferential attachment)* For a tolerant society, preferential attachment emerges if all agents are opportunistic, i.e. $\gamma_k = 1, \forall k \in \Theta$. □

In the following Corollary, we show that agents' ages in the network create inequality in the popularity capital.

Corollary 4. *(Superiority of older agents in tolerant societies)* For a tolerant society, we have that $\deg_i^-(t) \succeq \deg_j^-(t)$ for all $i < j$. □

Thus in the setting of Corollary 4, age is the only factor that creates inequality in popularity capital. In the following Corollary, we show that opportunism creates long term popularity advantages for agents forming the network.

Corollary 5. *(Opportunism is good in the long-run)* If $d_i^1(t)$ is the popularity of agent i at time t in a tolerant society with $\gamma_k = 0, \forall k \in \Theta$, and $d_i^2(t)$ is the popularity of agent i at time t in a tolerant society with $\gamma_k = 1, \forall k \in \Theta$, then we have that $\mathbb{E}[d_i^2(t)] \leq \mathbb{E}[d_i^1(t)]$ for all $t \leq T^*$, and $\mathbb{E}[d_i^2(t)] \geq \mathbb{E}[d_i^1(t)]$ for all $t > T^*$, where $T^* \leq i \times \left(-\bar{L}\,\mathcal{W}_{-1}\left(\frac{-1}{\bar{L}}e^{\frac{-1}{\bar{L}}}\right)\right)^{\frac{1}{b}}$, $b = \sum_{m\in\Theta} p_m L_m^*(0)$, and $\mathcal{W}_{-1}(.)$ is the lower branch of the Lambert W function [56]. □

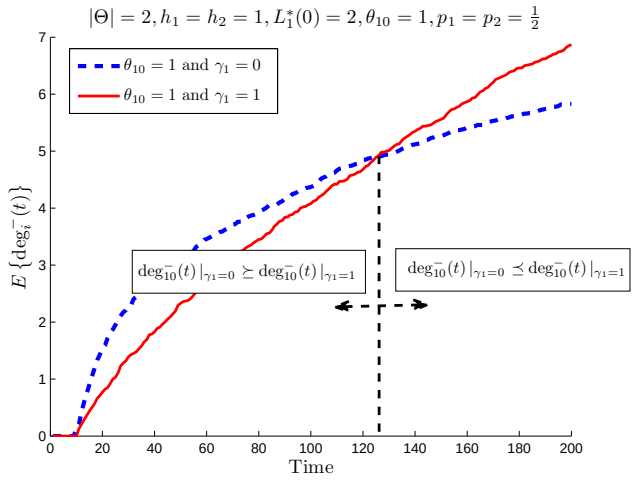


Fig. 5: Expected popularity over time for an agent entering at $t = 10$ for different levels of opportunism.

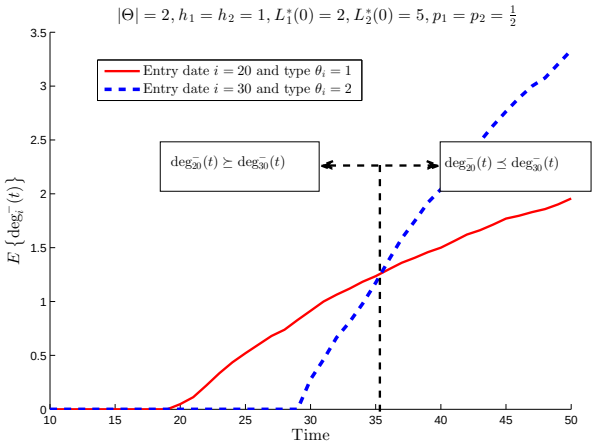


Fig. 6: Expected popularity over time for two agents belonging to 2 social categories with different levels of gregariousness.

Thus, in societies where individuals are opportunistic, the long-term popularity capital is harvested after a certain time threshold as shown in Fig. 5. Such threshold is increasing in the agents' average gregariousness. Thus, younger agents or agents in a society with large average gregariousness need to wait longer to harvest the popularity gains attained by opportunism. To sum up, in tolerant societies, only age creates popularity capital inequality, and the growth of the popularity capital is governed by both the level of opportunism and the average gregariousness of the agents' types. However, there is no inequality in the acquisition of popularity capital across different social category.

5.2 Popularity capital in intolerant societies

We now study popularity capital in societies for which $h_k = 1$ for all k; it seems natural to refer to such societies as intolerant (rather than totally homophilic). In the following Theorem, we study the popularity growth rates for different types of agents in the network.

Theorem 4. (Popularity growth in intolerant societies) For an intolerant society, the popularity growth rates are given as follows:

- For $\gamma_k = 0, \forall k \in \Theta$, the expected popularity of every agent i grows logarithmically with time, i.e. $\mathbb{E}[\deg_i^-(t) \,|\, \theta_i]$ is $O\left(L^*_{\theta_i}(0)\, \log(t)\right)$.
- For $\gamma_k = 1, \forall k \in \Theta$, the expected popularity of every agent i grows at least sublinearly with time, i.e. $\mathbb{E}[\deg_i^-(t) \,|\, \theta_i]$ is at least as fast as $O\left(t^{b_{\theta_i}}\right)$, where $b_k > b_m$ if $L^*_k(0) > L^*_m(0), \forall k, m \in \Theta$. □

Thus, for tolerant and intolerant societies, the popularity growth rates are qualititatively similar – but the sublinear growth obeys a different exponent. In the following Corollary, we show that gregariousness and opportunism create inequality in the popularity capital.

Corollary 6. *(Gregariousness and opportunism create inequality in the popularity capital)* For an intolerant society, and for the two agent types $k, m \in \Theta$ in the network with arbitrary p_k and p_m, the following is satisfied:

- If $\gamma_k = \gamma_m$, $\gamma_k \in \{0, 1\}$, and $L^*_k(0) > L^*_m(0)$, then there exists a time $T^* < \infty$ where $\mathbb{E}[\deg_i^-(t)] \geq \mathbb{E}[\deg_j^-(t)]$, for all $t > T^*$, where $\theta_i = k$ and $\theta_j = m$.
- If $\gamma_k = 1$ and $\gamma_m = 0$, and $L^*_k(0) = L^*_m(0)$, then there exists a time $T^* < \infty$ where $\mathbb{E}[\deg_i^-(t)] \geq \mathbb{E}[\deg_j^-(t)]$, for all $t > T^*$, where $\theta_i = k$ and $\theta_j = m$. □

This agent-level characterization can be further generalized to the collective popularity of social categories in the following Theorem; we show that the popularity distribution of a more gregarious (or opportunistic) social category stochastically dominates that of a less gregarious (or opportunistic) category.

Theorem 5. (Popularity capital inequality across social categories) For an intolerant society, the following is satisfied:

- If $\gamma_k = \gamma_m$, $\gamma_k \in \{0, 1\}$, and $L^*_k(0) > L^*_m(0)$, then $f_d^{t,k}(d)$ first order stochastically dominates $f_d^{t,m}(d)$ assuming a mean-field approximation for the popularity acquisition process.
- If $\gamma_k = 1$ and $\gamma_m = 0$ and $L^*_k(0) = L^*_m(0)$, then $f_d^{t,k}(d)$ first order stochastically dominates $f_d^{t,m}(d)$ assuming a mean-field approximation for the popularity acquisition process. □

Thus, for intolerant societies, popularity is influenced by gregariousness and structural opportunism rather than by population share. See Fig. 6. In contrast with tolerant societies, a younger agent in an intolerant society can become and remain more popular than an older agent if the younger agent belongs to a more gregarious or more opportunistic social category.

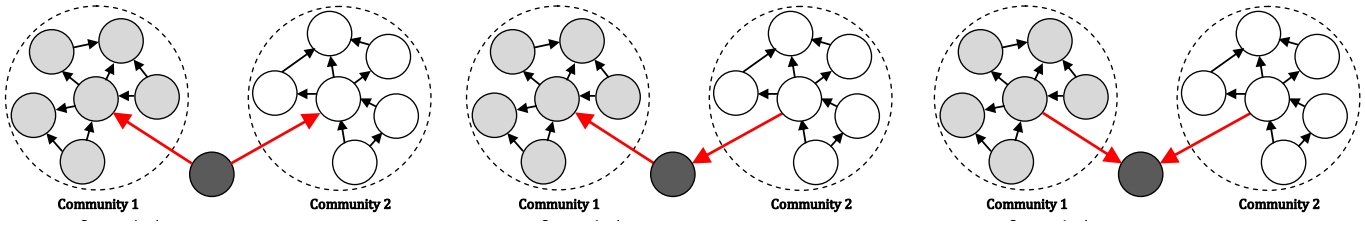
Theorem 5 can also be understood in the context of citation networks [57]. In the context of citation networks, intolerance means simply that papers only cite papers that are really related – which is of course very common and not at all unusual. [57] shows, in many scientific fields, that there is a strong positive correlation between the number of references per paper and the total number of citations. We quote the following conclusion from the report in [58], which is based on a statistical analysis of *Thomson Reuters' Essential Science Indicators* database: "*One might think that the number of papers published or the population of researchers in a field are the predominant factors that influence the average rate of citation, but it is mostly the average number of references presented in papers of the field that determines the average citation rate.*" This conclusion is in perfect agreement with Theorem 5 (and Corollary 6), which predict that for the inherently intolerant citation networks, the popularity of researchers in different fields (total citation rate) is governed by their "gregariousness" (number of references per paper), and not by the type distribution (number of papers/researchers). We know from [58] that papers in mathematics typically list few references, while those in molecular biology typically list many. Thus, molecular biologists are more "gregarious" than mathematicians – and one would expect that younger molecular biologists can, on average, become more "popular" – have higher citation indexes – than mathematicians, solely because of the differences between disciplines and entirely unrelated to "quality" or "real impact". This would seem to provide caution for University review committees.

Of course, other dimensions in addition to popularity/citation counts, express the value of scholarly work. One of these dimensions is *interdisciplinarity* [63], which is a form of bridging capital rather than a popularity capital as we show in the next Section.

6 Bridging Capital

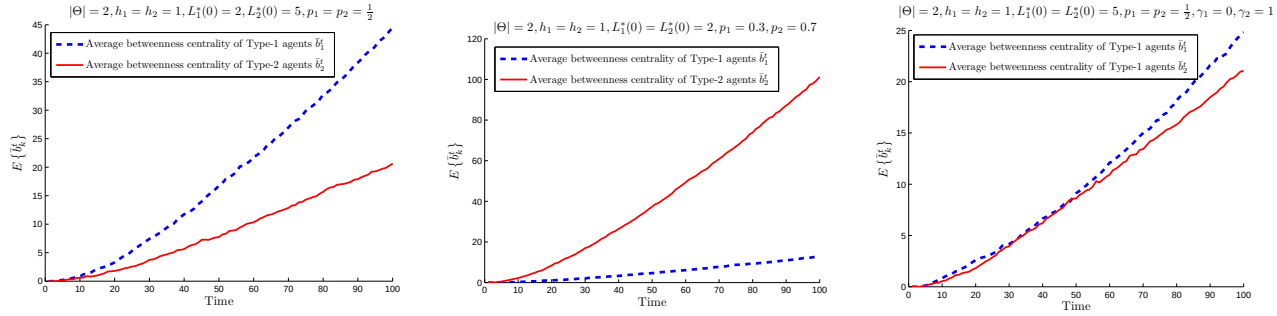
6.1 Betweenness centrality as a measure for bridging capital

In Sections 4 and 5, we have studied two forms of capital that share two basic features: they are egocentric in the sense that they create value for individuals, and they are only affected by the agents' local network structures. Bonding occurs when an individual socializes with similar individuals driven by homophily, whereas bridging occurs when an individual links multiple segregated communities. While bonding creates egocentric values for individuals, bridging creates shared value for the network, e.g. allows diverse research communities to exchange ideas and innovations. As Burt points out in [8], individuals with bridging capital enjoy a *central* position in the network as they act as a gateway for information exchange. Betweenness centrality, a graph-theoretic measure promoted by Freeman in [12], is a conventional measure for centrality since for a given agent it counts the number of shortest paths between any two agents that involves that agents, and thus reflects the agents' ability to broker interactions at the interface between different categories [60]- [64]. The betweenness centrality of



(a) Bridging via information dissemination. (b) Bridging via information transfer. (c) Bridging via information fusion.

Fig. 7: Different bridging modes with the same betweenness centrality for a central agent in a segregated network.



(a) Impact of gergariousness on agents' centrality. (b) Impact of type distribution on agents' centrality. (c) Impact of structural opportunism on agents' centrality.

Fig. 8: Estimates for the average betweenness centrality of two types of agents in an extremely homophilic agents.

agent i at time t, which is denoted by b_i^t, is an indicator of its centrality in the network [12], and is given by

$$b_i^t = \sum_{k \neq j \neq i} \frac{\sigma_{kj}(i)}{\sigma_{kj}}, \tag{8}$$

where σ_{kj} is the total number of shortest paths between k and j in G^t ignoring the edge directions, and $\sigma_{kj}(i)$ is the number of such paths that pass through i. In order to characterize the centrality of a certain social category, we define the average betweenness centrality of type-k agents $\bar{b}_k^t$ as follows

$$\bar{b}_k^t = \frac{1}{|\mathcal{V}_k^t|} \sum_{i \in \mathcal{V}_k^t} b_i^t.$$

Betweenness is a relational measure: an agent with a high betweenness centrality score does not belong to one of the dense groups, but relates them. While the evolving network is modeled as a directed graph, we capture the bridging capital by computing the betweenness centrality of agents in the *simplified* undirected version of the graph G^t. This is because bridging capital reflects the *structural centrality* of the agent, i.e. to what extent an agent is "between" segregated social groups, whereas the edge directions reflect the directions of information flow. As shown in Fig. 7, a central agent can either disseminate information to segregated groups, transfer information from one group to another, or gather information produced by different groups. In Fig. 7, the central agent has the same betweenness centrality score in the networks (a), (b), and (c), yet the role played by that agent in each network is different. In (a), the central agent gets non-redundant information from community 1 and community 2, which allows that agent to come up with *innovations and new ideas*. In (b), the central agent transfers information from community 1 to community 2, which allows that agent to *control* the flow of information across groups. In (c), the central agent displays *influence* on community 1 and community 2 by disseminating information to those communities. In the three networks, the bridging capital (i.e. extent of the agent's betweenness) is the same, yet the role of the central agent and the nature of its social advantage is different. We are interested in characterizing the extent of structural centrality of the agents in the network rather than the specific roles they play at the interface between groups.

Characterizing the betweenness centrality for a general network is not mathematically tractable, and only empirical and simulation results are obtained in the literature [60]-[61]. We start by presenting simulation results for the betweenness centrality of agents in a network with 2 types, and show the impact of the exogenous parameters. In Fig. 8, we plot estimates for the expected average betweenness centrality of 2 types of extremely homophilic agents obtained via a Monte Carlo simulation, highlighting the impact of gregariousness, type distribution, and structural opportunism. In Fig. 8(a), we can see that increasing gregariousness decreases centrality, which is intuitive since when each agent forms many links, the number of shortest paths that involve far agents in terms of the geodesic distance will decrease, which leads to a decrease in the average centrality of the whole social category. That is, when all agents are sociable, then all agent are less central (on average). This is in striking contrast with the popularity capital, where gregariousness of agents in a social category was helping them acquiring popularity. Moreover, we can see in Fig. 8(b) that the type distribution plays a role in determining the agents' centrality; majorities are more central than minorities. Such result, which agrees with the qualitative study of Ibarra in

[65], is again in a striking contrast with the popularity capital acquisition where the type distribution had no significant impact on the agents' popularity growth rates. Finally, Fig. 8(c) shows that structural opportunism decreases centrality, which is again in contrast with the popularity acquisition experience where structural opportunism was allowing for the emergence of preferential attachment.

From the simulation results in Fig. 8, we conclude that *homophily creates inequality in the acquisition of bridging capital*, and the different behaviors and norms of different social categories lead to the emergence of different forms of capital. The way that inequality is created in those forms of capital can have very different dependencies on the behaviors of the social categories. When agents in a homophilic group are very sociable, every agent is likely to be popular but not central. That is, socialization increases the bonding capital, but decreases the bridging capital. Moreover, minorities have the same chance as majorities to become popular, yet they have less chances to be central.

The results in Fig. 8 and the discussion above are concerned with the centrality of agents within their social groups. However, a more interesting form of centrality is the one that arises from bridging heterogeneous social categories. In fact, this is the form of social capital that Burt and Putnam have extensively studied in [6] and [8]. In the following subsection, we introduce a new phenomenon that provides insights into the interplay between centrality and homophily.

6.2 Homophily and intergroup bridging

In this subsection, we study a striking phenomenon that arises from the interplay between homophily and centrality. In particular, we show via simulations that when a social category possesses different homophilic tendencies compared to all other social categories, they end up being the most central group, and thus accrue the largest bridging capital. That is, in an extremely homophilic society, non-homophilic agents bridge segregated social groups, and thus become the most central and gain access to diverse sources of information. On the other hand, a homophilic social category in a non-homophilic society ends up being the most central as they form a highly connected core of the global network structure, which represent an information hub through which all individuals are bridged.

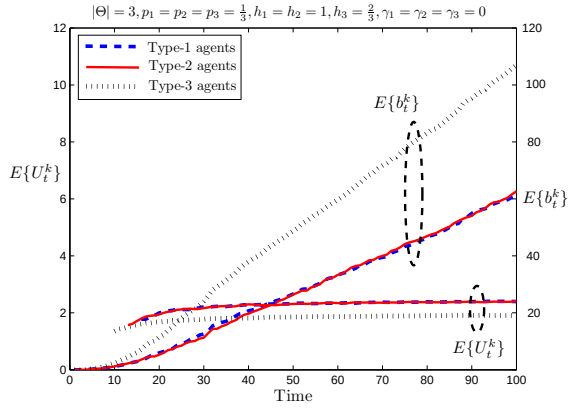
6.2.1 Filling structural holes: the power of tolerance, open-mindedness, and interdisciplinarity

As first pointed out by Granovetter in [7], weak ties (the ties between individuals of different types) have strength as they bridge different segregated social groups. Opinions, beliefs, and ideas are more homogeneous within than between groups, so individuals connected across groups are more exposed to alternative ways of thinking and behaving. In other words, brokerage across the structural holes between homophilic categories provides a vision of new options that are otherwise unseen, which stimulates new ideas and innovation, and also allows agents to control information flow across different groups [9]. Thus brokerage creates a social capital, namely a bridging capital, and centrality in such case is gained by agents who link the segregated homophilic groups.

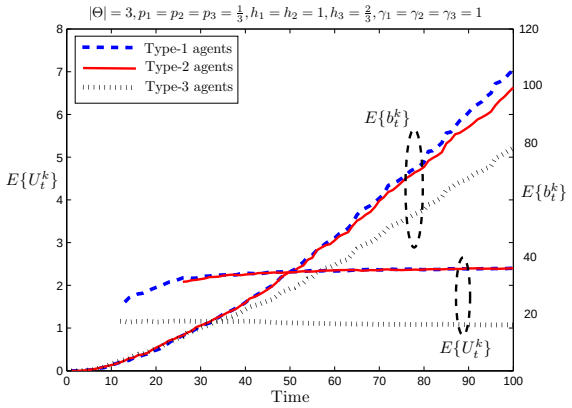
In Fig. 9, we demonstrate the interplay between bonding and bridging capital in a network that exhibits structural holes. We carry out a Monte Carlo simulation by simulating 1000 instantiations of the network and plot the average utility and betweenness centrality of each type of agents in the network. We assume that the network has 3 types of agents, where type-1 and type-2 agents are extremely homophilic, whereas type-3 agents have a homophily index of $h_3 = \frac{1}{3}$. Since type-1 and type-2 agents are extremely homophilic, their bonding capital is maximized, yet both types are disconnected, which creates an opportunity for harvesting bridging capital by type-3 agents since such a type can bridge the two disconnected communities. The ability of type-3 agents to acquire bridging capital depends on their meeting process, i.e. the extent to which they explore the network. If non-homophilic individuals are not exploring the network, then they will end up in a peripheral position in the network, and may not construct their ego networks in finite time (recall Lemma 1). Fig. 9(a) and 9(b) depict the impact of the meeting process on the bridging capital acquired by non-homophilic agents in a homophilic society. It is clear from both figures that there is a tension between the bonding capital (expressed in terms of the average utility), and the bridging capital (expressed in terms of the average centrality). That is to say, homophilic type-1 and type-2 agents acquire higher utility since they enjoy more homogeneous ego networks than type-3 agents. However, when $\gamma_3 = 0$, type-3 agents are more central in the network as they broker the interface between type-1 and type-2 social categories. Contrarily, when $\gamma_3 = 1$, type-3 agents acquire less bonding and bridging capital as they do not explore the network, thus they cannot bridge segregated groups, albeit being non-homophilic.

Fig. 10 depicts the network structure at $t = 1000$ for various meeting processes. In Fig. 10(a), we see that when type-3 agents (red colored) are fully opportunistic, they end up being either marginalized (acquire a peripheral position) or unsatisfied (never forms a satisfactory ego network). When the network exploration rate increases, we see in Fig. 10(b) that only a fraction of non-homophilic agents are peripheral at any time step, yet an intermediate community of such agents emerges and it bridges the otherwise segregated social groups. When $\gamma_3 = 1$, we see in Fig. 10(c) that all non-homophilic agents will reside in the central community and will acquire a central position. Such result provides the following interesting insight: it is not enough for individuals to be non-homophilic, tolerant, or open-minded in order to harvest the bridging capital, but it is essential for them to explore the network structure such that they meet diverse types of agents. Thus, in a society where the meeting process; reflected by policies, norms, regulations, geographical constraints or rules; hinders network exploration, then the existence of non-homophilic individuals does not guarantee that structural holes will be filled. In the following Theorem, we provide the necessary and sufficient conditions for any network to be connected.

Theorem 6. (Network connectedness) An asymptotically large network is connected almost surely, i.e. $\mathbb{P}\left(\lim_{t\to\infty} \omega\left(G^t\right) = 1\right) = 1$, if and only if there exists at least one type of agents k with $h_k < 1$ and $\gamma_k < 1$. □

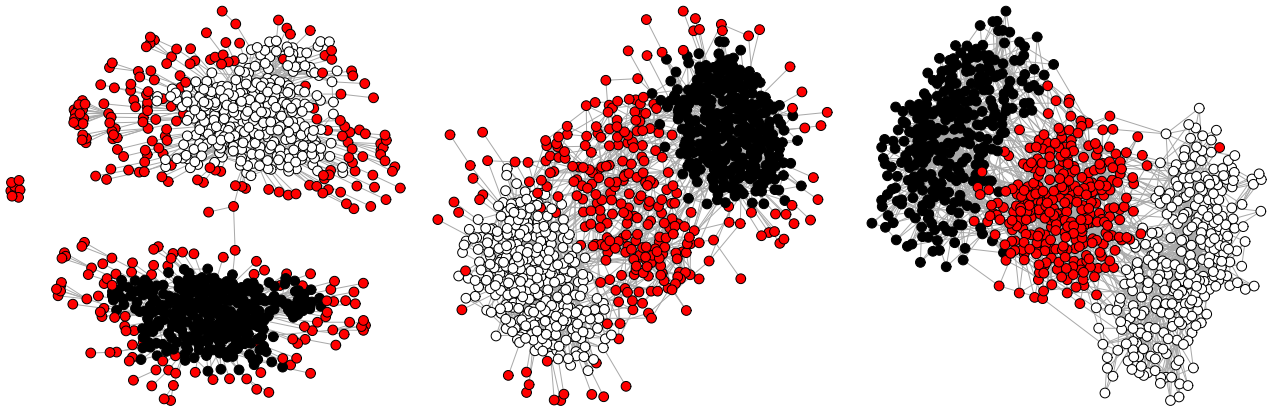


(a) Centrality and utility of non-homophilic and exploring agents in a homophilic society.



(b) Centrality and utility of non-homophilic, non-exploring agents in a homophilic society.

Fig. 9: Betweenness centrality and average utility of agents in a network that exhibits structural holes.



(a) $\gamma_3 = 1$. (b) $\gamma_3 = 0.1$. (c) $\gamma_3 = 0$..

Fig. 10: A Snapshot for a network with $h_1 = h_2 = 1$, $\gamma_1 = \gamma_2 = 1$, and $h_3 = \frac{1}{3}$. Non-homophilic agents acquire a central position in the network when they are less opportunistic.

Theorem 6 says that the existence of non-homophilic type of agents that explore the network with any non-zero rate will guarantee network connectedness. The condition of $\gamma_k < 1$ follows from our assumption that agents have infinite lifetimes. If agents have finite lifetimes, then a threshold on γ_k will decide the network connectedness. That is to say, open-minded individuals will have a threshold on the minimum rate of network exploration that is a function of their lifetime, beyond which they will not be able to fill the structural holes and acquire the largest bridging capital. Thus, non-homophilic agents, who can be thought of as being "tolerant" or "open-minded" individuals, can bridge segregated social groups and become the most central in the network when their meeting process involves exploring the network.

The literature argues that the centrality of non-homophilic (or tolerant and open-minded) individuals play an important role in many networks. For instance, in the context of citation networks, Leydesdorff proposes betweenness centrality as a measure of a journal's "interdisciplinarity". In addition to the *impact factor* which is a measure of a journal's influence, centrality of a journal indicates the role it plays in promoting innovative and interdisciplinary research, which creates a social capital in the research citation and collaboration networks [63] [64]. Moreover, Burt emphasizes the role of centrality in the diffusion of information [8], and the creation of new ideas as a result to the exposure to non redundant sources of information [9]. It is worth noting that bridging capital not only leads to egocentric returns to individuals, but also creates a shared value for the network: it stimulates innovative and interdisciplinary research ideas, and allows for the diffusion of information along the global network structure.

6.2.2 *Emergence of information hubs: the power of the dominant coalition*

In the previous subsection, we have shown that non-homophilic agents in a homophilic society acquire the most central network positions and thus attain the highest bridging capital. In this subsection, we show that in the reciprocal scenario where there is one homophilic type of agents in a non-homophilic society, homophilic agents end up being more central than others. In Fig. 11, we plot the

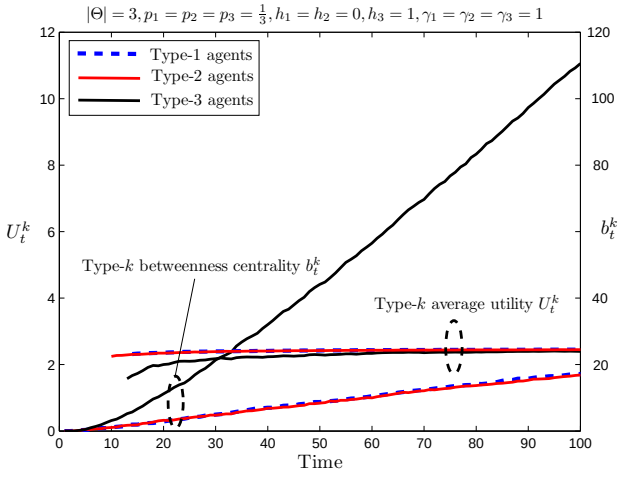


Fig. 11: Betweenness centrality and average utility of agents in a network with a dominant information hub.

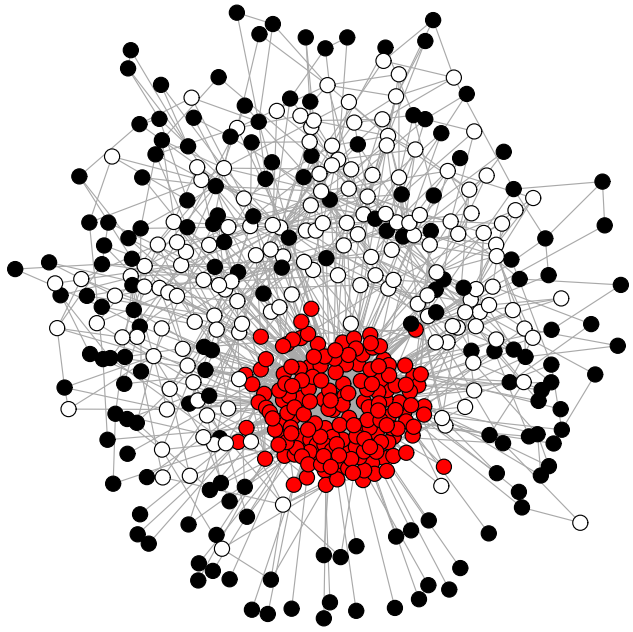


Fig. 12: The formation of an information hub in a network with $h_1 = h_2 = \frac{1}{3}$, and $h_3 = 1$. The extremely homophilic type-3 agents form a dominant coalition that resides in the core of the network.

average utility and betweenness centrality of 3 types of agents forming a network, where types 1 and 2 agents are extremely non-homophilic, whereas type 3 agents are extremely homophilic. It can be observed that the average centrality of type 3 agents dominates that of types 1 and 2. This is because type 3 agents tend to connect to each other, thus forming a *dominant coalition* or an *information hub* that resides in the core of the network, and acts as a central "super-node" in a star-like graph, hence achieving a high level of centrality (see Appendix M for an illustration). The term "dominant coalition" was coined by Brass in [67] to describe same-gender highly connected influential agents in an organization's interaction network. Unlike the result of the previous subsection, homophilic central agents in a society dominated by non-homophilic types of agents do not bridge structural holes in the network, but rather form a densely connected sub-network through which information is disseminated over the entire network topology. In the context of citation networks, this result predicts that if types corresponds to journals, then a journal that is highly cited and at the same time maintains a self-citation rate that is significantly higher than other journals is likely to form an information hub in a network of papers. Fig. 12 illustrates the formation of an information hub by the extremely homophilic agents in a non-homophilic society, where it can be seen that the type-3 agents form a core sub-network that resides in the center of the global network topology.

7 Conclusions

In this paper, we presented a micro-founded mathematical model of the emerging social capital in evolving social networks. In our model, the evolution of the network and of social capital are driven by exogenous and endogenous processes, which are influenced by the extent to which individuals are homophilic, structurally opportunistic, socially gregarious and by the distribution of agents' types in the society. We focused on three different forms of endogenously emerging social capital: bonding, popularity, and bridging capital, and showed how these different forms of capital depend on the exogenous parameters. Bonding capital is maximized in extremely homophilic societies, yet extreme homophily creates structural holes that hinder communications across network components. Popularity capital leads to preferential attachment due to the agents' structural opportunism, which offers agents a cumulative advantage in popularity capital acquisition. Homophily creates inequality in the popularity capital; more gregarious types of agents are more likely to become popular. However, in homophilic societies, individuals who belong to less gregarious, less opportunistic, or major types are likely to be more central in the network and thus acquire a bridging capital. Finally, we studied a striking phenomenon that arises from the interplay between homophily and centrality. In particular, we showed that when a social category that possesses different homophilic tendencies compared to all other social categories, they end up being the most central group, and thus accrue the largest bridging capital. Future research directions may include studying a network with foresighted agents, and investigating the impact of an initial network construction on the emerging network's growth paths.

APPENDIX A
PROOF OF LEMMA 1

Our goal is to prove that an agent i is socially unsatisfied with a positive probability, i.e. $\mathbb{P}(T_i = \infty \,|\theta_i) > 0$, if and only if $\gamma_{\theta_i} = 1$ and $0 < h_{\theta_i} < 1$. That is, we want to prove that the following statement is true:

$$(\gamma_{\theta_i} = 1) \wedge (0 < h_{\theta_i} < 1) \iff \mathbb{P}(T_i = \infty \,|\theta_i) > 0. \quad \text{(A.1)}$$

Proving that statement (A.1) is true requires proving the truth of the following *sufficiency* and *necessity* statements:

Sufficiency: $(\gamma_{\theta_i} = 1) \wedge (0 < h_{\theta_i} < 1) \Rightarrow \mathbb{P}(T_i = \infty \,|\theta_i) > 0.$

Necessity: $(\gamma_{\theta_i} = 1) \wedge (0 < h_{\theta_i} < 1) \Leftarrow \mathbb{P}(T_i = \infty \,|\theta_i) > 0.$

We start by proving the sufficiency condition in Subsection A.1, and then we prove the necessity condition in Subsection A.2, thereby concluding the proof of the lemma. Before proceeding with the proofs, we provide a useful Lemma which will be used in Subsection A.1.

Lemma A.1. *For $\gamma_{\theta_i} = 1$ and $h_{\theta_i} \in [0, 1]$, we have that $\mathbb{P}(N_i^s(\infty) = L^*_{\theta_i}(0) \,|\, \theta_i) > 0$.*

Proof We can prove that $\mathbb{P}(N_i^s(\infty) = L^*_{\theta_i}(0) \,|\, \theta_i) > 0$ by showing that the event that agent i meets $L^*_{\theta_i}(0)$ similar-type agents consecutively upon entering the network, i.e. the event $\{\theta_{m_i(t)} = \theta_i\}_{t=i}^{i+L^*_{\theta_i}(0)-1}$, happens with a positive probability. Note that the probability that agent i meets a similar-type agent at any given time step t given a step graph G^t is (refer to Section 3.3)

$$\mathbb{P}(\theta_{m_i(t)} = \theta_i \,|\, G^t) = \begin{cases} \frac{|\mathcal{K}^{\theta_i}_{i,t}|}{|\mathcal{K}_{i,t}|}, & \deg_i^+(t) > 0,\ \mathcal{K}_{i,t} \neq \emptyset, \\ \frac{|\mathcal{V}^t_{\theta_i}|}{t}, & \text{Otherwise}, \end{cases} \quad \text{(A.2)}$$

which is always non-zero whenever $|\mathcal{K}^{\theta_i}_{i,t}| > 0$ and $|\mathcal{V}^t_{\theta_i}| > 0$. In (A.3), we derive the probability of the event $\{\theta_{m_i(t)} = \theta_i\}_{t=i}^{M}$, where $M = i + L^*_{\theta_i}(0) - 1$, in terms of the conditional probability $\mathbb{P}(\theta_{m_i(t)} = \theta_i \,|\, G^t)$ in (A.2) using Bayes' rule. Since we know from (A.2) that the term $\mathbb{P}(\theta_{m_i(t)} = \theta_i \,|\, G^t)$ in (A.3) is non-zero for $|\mathcal{K}^{\theta_i}_{i,t}| > 0$, and since the random variable $|\mathcal{K}^{\theta_i}_{i,t}|$ is non-degenerate, then it follows that the terms $\sum_{G^t} \mathbb{P}(\theta_{m_i(t)} = \theta_i \,|\, G^t)\, \mathbb{P}(G^t \,|\, G^i, \{\theta_{m_i(j)} = \theta_i\}_{j=i}^{t-1})$ in (A.3) are non-zero, and hence we have that $\mathbb{P}(\{\theta_{m_i(t)} = \theta_i\}_{t=i}^{M}) > 0$, which concludes the proof of the Lemma. □

A.1 Proving Sufficiency

In this Subsection, we want to prove that

$$(\gamma_{\theta_i} = 1) \wedge (0 < h_{\theta_i} < 1) \Rightarrow \mathbb{P}(T_i = \infty \,|\theta_i) > 0. \quad \text{(A.4)}$$

We start by examining the meeting process and the linking actions of an agent i, with a type θ_i that satisfies the conditions $\gamma_{\theta_i} = 1$ and $0 < h_{\theta_i} < 1$. For such an agent, the following events take place over time:

- **At time $t = i$:** Agent i enters the network, meets agent $m_i(i)$, and since $h_{\theta_i} < 1$, it links to that agent irrespective of its type, i.e. $a^i_i = 1$ for any $\theta_{m_i(i)} \in \Theta$.
- **At time $t > i$:** Since $\gamma_{\theta_i} = 1$, agent i meets an agent $m_i(t)$ picked randomly from the choice set $\mathcal{K}_{i,t}$, and decides whether or not to form a link with that agent based on its type.

We know from Section 3.5 that since $h_i > 0$, then agent i has to connect to at least one agent of type θ_i in order to get socially satisfied. If agent i never meets an agent of type θ_i, then $T_i = \infty$. Combining this fact with the meeting process described above, we have that

$$(\theta_{m_i(i)} \neq \theta_i) \wedge (\cup_{t=i+1}^{\infty} \mathcal{K}^{\theta_i}_{i,t} = \emptyset) \Rightarrow (T_i = \infty), \quad \text{(A.5)}$$

where $\mathcal{K}^{\theta_i}_{i,t} \subseteq \mathcal{K}_{i,t}$ is the set of type-θ_i followees of followees for agent i at time t. Statement (A.5) says that if agent i does not meet a type-θ_i agent at time step i, i.e. $(\theta_{m_i(i)} \neq \theta_i)$, and if the choice set $\mathcal{K}_{i,t}$ is free of type-θ_i agents for all time steps $t > i$, i.e. $(\cup_{t=i+1}^{\infty} \mathcal{K}^{\theta_i}_{i,t} = \emptyset)$, then agent i will be socially unsatisfied, i.e. $T_i = \infty$. It follows from statement (A.5) that

$$\mathbb{P}(T_i = \infty \,|\theta_i) \geq \mathbb{P}(\theta_{m_i(i)} \neq \theta_i, \cup_{t=i+1}^{\infty} \mathcal{K}^{\theta_i}_{i,t} = \emptyset \,|\, \theta_i). \quad \text{(A.6)}$$

Thus, it suffices to show that $\mathbb{P}(\theta_{m_i(i)} \neq \theta_i, \cup_{t=i+1}^{\infty} \mathcal{K}^{\theta_i}_{i,t} = \emptyset \,|\, \theta_i)$ is bounded away from zero in order to prove the sufficiency condition in (A.4). Note that using Bayes' theorem, the right hand side in (A.6) can be decomposed as follows

$$\mathbb{P}(\theta_{m_i(i)} \neq \theta_i, \cup_{t=i+1}^{\infty} \mathcal{K}^{\theta_i}_{i,t} = \emptyset \,|\, \theta_i) = \mathbb{P}(\cup_{t=i+1}^{\infty} \mathcal{K}^{\theta_i}_{i,t} = \emptyset \,|\, \theta_{m_i(i)} \neq \theta_i, \theta_i)\, \mathbb{P}(\theta_{m_i(i)} \neq \theta_i \,|\, \theta_i). \quad \text{(A.7)}$$

Since $\mathbb{P}(\theta_{m_i(i)} \neq \theta_i \,|\, \theta_i) = 1 - p_{\theta_i} > 0$, then we only need to show that $\mathbb{P}(\cup_{t=i+1}^{\infty} \mathcal{K}^{\theta_i}_{i,t} = \emptyset \,|\, \theta_{m_i(i)} \neq \theta_i, \theta_i)$ is strictly positive in order to conclude the sufficiency condition in (A.4). Hence, in the rest of this Subsection, we will prove that

$$\mathbb{P}(\cup_{t=i+1}^{\infty} \mathcal{K}^{\theta_i}_{i,t} = \emptyset \,|\, \theta_{m_i(i)} \neq \theta_i, \theta_i) > 0. \quad \text{(A.8)}$$

Direct evaluation of the probability in (A.8) is a hard task, and hence we first obtain a lower bound on $\mathbb{P}(\cup_{t=i+1}^{\infty} \mathcal{K}^{\theta_i}_{i,t} = \emptyset \,|\, \theta_{m_i(i)} \neq \theta_i, \theta_i)$, and then show that such a lower bound is bounded away from zero. We achieve this by establishing a chain of simple conditions that logically suffice for condition $(\cup_{t=i+1}^{\infty} \mathcal{K}^{\theta_i}_{i,t} = \emptyset)$ in (A.8) to be true. To that end, we define the conditions $\mathbb{C}1$, $\mathbb{C}2$, and $\mathbb{C}3$ as follows.

$$\mathbb{C}1 : \cup_{t=i+1}^{\infty} \mathcal{K}^{\theta_i}_{i,t} = \emptyset.$$
$$\mathbb{C}2 : \cup_{t=i+1}^{\infty} \mathcal{N}^{+,\theta_i}_{m_i(t),t} = \emptyset.$$
$$\mathbb{C}3 : \wedge_{t=i}^{i+\bar{L}^*_{\theta_i}(0)-1} (N^s_{m_i(t)}(t) = L^*_{\theta_{m_i(t)}}(0)).$$

That is, $\mathbb{C}1$ is the condition that agent i has no type-θ_i agents in its choice set $\mathcal{K}_{i,t}$ for all $t > i$, $\mathbb{C}2$ is the condition that none of the agents that agent i meets at $t > i$ have a type-θ_i agent in their followee sets, and $\mathbb{C}3$ is the condition that each of the first $\bar{L}^*_{\theta_i}(0)$ agents[3] that agent i meets are linked only to same-type agents. Whenever $(\theta_{m_i(i)} \neq \theta_i)$ is true, the following statements hold:

$$\mathbb{C}2 \overset{\text{(a)}}{\iff} \mathbb{C}1$$
$$\mathbb{C}3 \overset{\text{(b)}}{\Rightarrow} \mathbb{C}2. \quad \text{(A.9)}$$

Statement (A.9)-a is tautological: it follows from the fact that agent i's choice set $\mathcal{K}_{i,t}$ is the union of the followee sets $\{\mathcal{N}^{+,\theta_i}_{m_i(t),t}\}_{m_i(t)}$ of the agents it meets. Statement (A.9)-b follows from the fact that if the agents $\{m_i(t)\}$ were

3. Refer to Section 3.5 for the definition of $\bar{L}^*_{\theta_i}(0)$.

$$\begin{aligned}\mathbb{P}(\{\theta_{m_i(t)}=\theta_i\}_{t=i}^{M}) &= \textstyle\sum_{G^i}\mathbb{P}(\{\theta_{m_i(t)}=\theta_i\}_{t=i}^{M}\,|\,G^i)\,\mathbb{P}(G^i)\\ &= \textstyle\sum_{G^i}\mathbb{P}(G^i)\prod_{t=1}^{M}\mathbb{P}(\theta_{m_i(t)}=\theta_i\,|\,G^i,\{\theta_{m_i(j)}=\theta_i\}_{j=i}^{t-1}).\\ &= \textstyle\sum_{G^i}\mathbb{P}(G^i)\prod_{t=1}^{M}\sum_{G^t}\mathbb{P}(\theta_{m_i(t)}=\theta_i\,|\,G^t)\,\mathbb{P}(G^t\,|\,G^i,\{\theta_{m_i(j)}=\theta_i\}_{j=i}^{t-1}).\end{aligned}\tag{A.3}$$

connected to other agents of their own type only, then their followee sets $\{\mathcal{N}^{+,\theta_i}_{m_i(t),t}\}_{m_i(t)}$ will not contain a type-θ_i agent. By forward chaining, (A.9) implies that $\mathbb{C}3 \Rightarrow \mathbb{C}1$, and hence we have that

$$\mathbb{P}(\mathbb{C}1\,|\,\theta_{m_i(i)}\neq\theta_i,\theta_i)\geq\mathbb{P}(\mathbb{C}3\,|\,\theta_{m_i(i)}\neq\theta_i,\theta_i).\tag{A.10}$$

Based on the inequality in (A.10), we can prove that inequality in (A.8) by showing that $\mathbb{P}(\mathbb{C}3\,|\,\theta_{m_i(i)}\neq\theta_i,\theta_i)$ is bounded away from zero, i.e. we need to show that

$$\mathbb{P}(\wedge_{t=i}^{i+\bar{L}^*_{\theta_i}(0)-1}(N^s_{m_i(t)}(t)=L^*_{\theta_{m_i(t)}}(0))\,|\,\theta_{m_i(i)}\neq\theta_i,\theta_i)>0.\tag{A.11}$$

From Lemma A.1, we know that the inequality in (A.11) holds. Combining (A.10) and (A.11), we conclude that the inequality in (A.8) also holds, which concludes the sufficiency condition in (A.4).

A.2 Proving Necessity

In this Subsection, we prove the necessity condition:

$$(\gamma_{\theta_i}=1)\wedge(0<h_{\theta_i}<1)\Leftarrow\mathbb{P}\left(T_i=\infty\,|\theta_i\right)>0,\tag{A.12}$$

which is the converse of the sufficiency condition in (A.4). We prove the statement in (A.12) by showing that when $\mathbb{P}\left(T_i=\infty\,|\theta_i\right)>0$, both the conditions $0<h_{\theta_i}<1$ and $\gamma_{\theta_i}=1$ must be satisfied.

Note that the statement in (A.12) can be broken down as follows

$$\begin{aligned}(0<h_{\theta_i}<1)&\Leftarrow\mathbb{P}\left(T_i=\infty\,|\theta_i\right)>0,\\ (\gamma_{\theta_i}=1)&\Leftarrow\mathbb{P}\left(T_i=\infty\,|\theta_i\right)>0.\end{aligned}\tag{A.13}$$

We first prove that $(0<h_{\theta_i}<1)\Leftarrow\mathbb{P}\left(T_i=\infty\,|\theta_i\right)>0$. Note that this statement can be further broken down into the following equivalent statements

$$\begin{aligned}(h_{\theta_i}>0)&\overset{\text{(a)}}{\Leftarrow}\mathbb{P}\left(T_i=\infty\,|\theta_i\right)>0,\\ (h_{\theta_i}<1)&\overset{\text{(b)}}{\Leftarrow}\mathbb{P}\left(T_i=\infty\,|\theta_i\right)>0.\end{aligned}\tag{A.14}$$

We start by proving statement (A.14)-a. In order for $\mathbb{P}\left(T_i=\infty\,|\theta_i\right)>0$ to be true, then $(T_i=\infty)$ must be true for a nonempty set of realizations of the graph process $\{G^t\}_t$. Assume that $(T_i=\infty)$ is true for a particular realization of $\{G^t\}_t$, then in such a realization agent i follows an action profile $(a^i_i,\ldots,a^t_i,\ldots)$ for which $a^j_i=0,\forall j>\bar{t}$ for some $\bar{t}\geq i$. That is, agent i deters from forming links with all the agents it meets after some time step $\bar{t}$. This can be true only if $L^*_{\theta_i}(\alpha^d_{\theta_i}\bar{L}^*_{\theta_i}(0))>0$, i.e. agent i must link to a similar-type followee in order to get socially satisfied. Since $h_{\theta_i}=\frac{L^*_{\theta_i}(\alpha^d_{\theta_i}\bar{L}^*_{\theta_i}(0))}{L^*_{\theta_i}(\alpha^d_{\theta_i}\bar{L}^*_{\theta_i}(0))+\bar{L}^*_{\theta_i}(0)}$ (see Section 3.5), then it follows that $h_{\theta_i}>0$.

Now we prove statement (A.14)-b by contradiction. That is, we assume that $(T_i=\infty)$ for some realization of $\{G^t\}_t$, and that the condition $h_{\theta_i}=1$ is true as well. Since $(T_i=\infty)$ is true, then agent i follows an action profile $(a^i_i,\ldots,a^t_i,\ldots)$ for which $a^j_i=0,\forall j>\bar{t}$ for some $\bar{t}\geq i$. This means that agent i never meets a similar-type agent after time step $\bar{t}$, i.e. $\mathbb{P}(\theta_{m_i(t)}=\theta_i)=0,\forall t>\bar{t}$. Note that the meeting process of agent i is given as follows (refer to Section 3.3):

$$\mathbb{P}(\theta_{m_i(t)}=\theta_i)=\begin{cases}\gamma_{\theta_i}+(1-\gamma_{\theta_i})\frac{|\mathcal{V}^t_{\theta_i}|}{t}, & \deg^+_i(t)>0\\ \frac{|\mathcal{V}^t_{\theta_i}|}{t}, & \deg^+_i(t)=0,\end{cases}\tag{A.15}$$

which for an asymptotically large t converges to $\gamma_{\theta_i}+(1-\gamma_{\theta_i})p_{\theta_i}$; since $\gamma_{\theta_i}+(1-\gamma_{\theta_i})p_{\theta_i}$ is strictly positive, no finite value for $\bar{t}$ satisfies that $\mathbb{P}(\theta_{m_i(t)}=\theta_i)=0,\forall t>\bar{t}$, which contradicts with $(T_i=\infty)$ being true. Therefore, in order for $(T_i=\infty)$ to be true, we must have that $h_{\theta_i}<1$, which concludes the proof of statement (A.14).

Now we prove the statement

$$(\gamma_{\theta_i}=1)\Leftarrow\mathbb{P}\left(T_i=\infty\,|\theta_i\right)>0$$

in (A.13). Again, we prove this statement by contradiction. Assume that both $(T_i=\infty)$ and $(\gamma_{\theta_i}<1)$ are true for some agent i in the network. For such an agent, the probability of meeting a similar-type agent is

$$\mathbb{P}(\theta_{m_i(t)}=\theta_i)=\begin{cases}\gamma_{\theta_i}\frac{|\mathcal{K}^{\theta_i}_{i,t}|}{|\mathcal{K}_{i,t}|}+(1-\gamma_{\theta_i})p_{\theta_i}, & \deg^+_i(t)>0\\ p_{\theta_i}, & \deg^+_i(t)=0,\end{cases}$$

which is always positive. Hence, any agent with $\gamma_{\theta_i}<1$ has a non-zero probability for meeting a similar-type agent at each time step, which means that such an agent is not socially unsatisfied in the almost sure sense. Thus, $\mathbb{P}\left(T_i=\infty\,|\theta_i\right)>0$ implies that $\gamma_{\theta_i}=1$.

APPENDIX B
PROOF OF THEOREM 1

We prove statements (1) and (2) in Subsections B.1 and B.2 respectively.

B.1 Proof of Statement (1)

Recall that the exogenous homophily index of an agent i is given by

$$h_{\theta_i}=\frac{L^*_{\theta_i}(\alpha^d_{\theta_i}\bar{L}^*_{\theta_i}(0))}{\bar{L}^*_{\theta_i}(0)+L^*_{\theta_i}(\alpha^d_{\theta_i}\bar{L}^*_{\theta_i}(0))}.\tag{B.1}$$

If $h_{\theta_i}=0$, then from (B.1) we know that $L^*_{\theta_i}(\alpha^d_{\theta_i}\bar{L}^*_{\theta_i}(0))=0$, and $L^*_{\theta_i}(0)=\bar{L}^*_{\theta_i}(0)$. Thus, it follows from (3) and (4) that

agent i forms a link with the first $L^*_{\theta_i}(0)$ agents it meets irrespective of their types, i.e.

$$\mathbb{P}(a_i^t = 1 \,|\, \theta_{m_i(t)}) = \begin{cases} 1, & t \leq L^*_{\theta_i}(0), \\ 0, & t > L^*_{\theta_i}(0), \end{cases}$$

and it follows that $\mathbb{P}\left(T_i = L^*_{\theta_i}(0)\right) = 1$.

B.2 Proof of Statement (2)

It follows from (B.1) that when $h_{\theta_i} = 1$, we have that $\bar{L}^*_{\theta_i}(0) = 0$, i.e. agent i forms links with similar-type agents only. From (3), we know that agent i forms exactly $L^*_{\theta_i}(0)$ links. The rest of the proof is organized as follows. First, we evaluate the probability distribution of the EFT T_i for agent i, and then we show that for a large network, this distribution becomes independent of the network topology. Next, we evaluate the expected EFT T_i.

The probability that agent i forms a link at a given time step t conditioned on the current step graph $\mathcal{G}_t$ and the realized meeting process is

$$\mathbb{P}(a_i^t = 1 \,|\, \theta_{m_i(t)}, G^t) = \mathbf{1}_{\left\{\theta_{m_i(t)} = \theta_i,\, \deg_i^+(t) \leq L^*_{\theta_i}(0)\right\}}. \quad \text{(B.2)}$$

That is, agent i forms a link with probability 1 if it meets a similar-type ($\theta_{m_i(t)} = \theta_i$) and it has not formed $L^*_{\theta_i}(0)$ links yet ($\deg_i^+(t) \leq L^*_{\theta_i}(0)$), and it forms a link with probability 0 otherwise. From the law of total probability, we can write the probability that agent i forms a link at time t conditioned on the step graph G^t as

$$\begin{aligned}\mathbb{P}(a_i^t = 1 \,|\, G^t) &= \mathbb{P}(\theta_{m_i(t)} = \theta_i \,|\, G^t)\, \mathbb{P}(a_i^t = 1 \,|\, \theta_{m_i(t)} = \theta_i, G^t) \\ &\quad + \mathbb{P}(\theta_{m_i(t)} \neq \theta_i \,|\, G^t)\, \mathbb{P}(a_i^t = 1 \,|\, \theta_{m_i(t)} \neq \theta_i, G^t).\end{aligned}$$

From (B.2), we know that

$$\mathbb{P}(a_i^t = 1 \,|\, \theta_{m_i(t)} \neq \theta_i, G^t) = 0,$$

and hence we can write $\mathbb{P}(a_i^t = 1 \,|\, G^t)$ as

$$\begin{aligned}\mathbb{P}(a_i^t = 1 \,|\, G^t) &= \mathbb{P}(\theta_{m_i(t)} = \theta_i \,|\, G^t)\, \mathbb{P}(a_i^t = 1 \,|\, \theta_{m_i(t)} = \theta_i, G^t) \\ &= \mathbb{P}\left(\theta_{m_i(t)} = \theta_i \,\middle|\, G^t\right)\, \mathbf{1}_{\left\{\deg_i^+(t) \leq L^*_{\theta_i}(0)\right\}}.\end{aligned} \quad \text{(B.3)}$$

The probability that agent i meets a similar-type agent, $\mathbb{P}\left(\theta_{m_i(t)} = \theta_i \,|G^t\right)$, is given by

$$\mathbb{P}(\theta_{m_i(t)} = \theta_i \,|\, \deg_i^+(t) = 0, G^t) = \frac{|\mathcal{V}^t_{\theta_i}| - 1}{|\mathcal{V}^t| - 1}, \quad \text{(B.4)}$$

and

$$\begin{aligned}&\mathbb{P}\left(\theta_{m_i(t)} = \theta_i \,\middle|\, \deg_i^+(t) > 0, G^t\right) = \\ &\left((1-\gamma_{\theta_i})(1 - \mathbb{P}(\mathcal{K}_{i,t} = \emptyset \,|\, G^t)) + \mathbb{P}(\mathcal{K}_{i,t} = \emptyset \,|\, G^t)\right) \times \\ &\mathbb{P}\left(\theta_{m_i(t)} = \theta_i \,\middle|\, m_i(t) \in \bar{\mathcal{K}}_{i,t} \cup \mathcal{K}_{i,t}, G^t\right) + \\ &\gamma_{\theta_i}(1 - \mathbb{P}(\mathcal{K}_{i,t} = \emptyset \,|\, G^t))\, \mathbb{P}(\theta_{m_i(t)} = \theta_i \,|\, m_i(t) \in \mathcal{K}_{i,t}, G^t),\end{aligned}$$

which can be simplified as follows

$$\begin{aligned}&\mathbb{P}(\theta_{m_i(t)} = \theta_i \,|\, \deg_i^+(t) > 0, G^t) = \\ &\frac{\gamma_{\theta_i} K_i^s(t)}{K_i(t)}\,(1 - \mathbb{P}(K_i(t) = 0 \,|\, G^t)) + \\ &(1 - \gamma_{\theta_i} + \gamma_{\theta_i}\mathbb{P}(K_i(t) = 0 \,|\, G^t))\frac{|\mathcal{V}^t_{\theta_i}| - N_i^s(t) - 1}{|\mathcal{V}^t| - N_i^s(t) - N_i^d(t) - 1}.\end{aligned} \quad \text{(B.5)}$$

The expressions in (B.4) and (B.5) follow directly from the meeting process described in Section 3.3. Since at time t agent i meets agents picked uniformly at random from the network when it has no followees by that time, then the probability of meeting similar-type agents in (B.4) is simply the fraction of type-θ_i agents in the network (after excluding agent i), i.e. $(|\mathcal{V}^t_{\theta_i}| - 1)/(|\mathcal{V}^t| - 1)$. If at time t agent i has at least one link (i.e. $\deg_i^+(t) > 0$), then it meets a stranger from the set $\bar{\mathcal{K}}_{i,t}$ with probability $(1 - \gamma_{\theta_i})$ (or if the set $\mathcal{K}_{i,t}$ is empty), or a followee of a followee from the set $\mathcal{K}_{i,t}$. As shown in (B.5), the fractions of type-θ_i agents in sets $\mathcal{K}_{i,t}$ and $\bar{\mathcal{K}}_{i,t}$ are $K_i^s(t)/K_i(t)$ and $(|\mathcal{V}^t_{\theta_i}| - N_i^s(t) - 1)/(|\mathcal{V}^t| - N_i^s(t) - N_i^d(t) - 1)$, respectively. By combining (B.3), (B.4) and (B.5), the probability of forming a link at any given time step t conditioned on the step graph G^t is given by (B.7). Now we show that for an asymptotically large network, the conditional probability in (B.7) becomes independent on the network topology G^t. Note that the probability that the set is empty, i.e. $\mathbb{P}(K_i(t) = 0 \,|\, G^t)$ in (B.5), can be bounded as follows

$$\mathbb{P}(K_i(t) = 0 \,|\, G^t) \leq \frac{1}{|\mathcal{V}^t_{\theta_i}|} \sum_{j \in \mathcal{V}^t_{\theta_i}} \mathbf{1}_{\{t < T_j\}}. \quad \text{(B.8)}$$

That is, the probability of the event that the followees of followees choice set of agent i becomes empty is always less than the probability of the event that agent i links to an agent j with less than $L^*_{\theta_i}(0)$ followees; this follows from the fact the occurrence of the first event implies that the second event has occurred. Since, $\lim_{t\to\infty} \frac{1}{|\mathcal{V}^t_{\theta_i}|} \sum_{j \in \mathcal{V}^t_{\theta_i}} \mathbf{1}_{\{t < T_j\}} = 0$, then if follows that $\mathbb{P}(K_i(t) = 0) \to 0$ in an asymptotically large network. Furthermore, since we know from Lemma 1 that agent i forms $L^*_{\theta_i}(0)$ links in a finite time almost surely, then we have that

$$\begin{aligned}\lim_{t\to\infty} \frac{|\mathcal{V}^t_{\theta_i}| - N_i^s(t) - 1}{|\mathcal{V}^t| - N_i^s(t) - N_i^d(t) - 1} &= \lim_{t\to\infty} \frac{p_{\theta_i} t - L^*_{\theta_i}(0) - 1}{t - L^*_{\theta_i}(0) - 1} \\ &= p_{\theta_i}.\end{aligned} \quad \text{(B.9)}$$

This leads to the expressions in (B.8), which implies that for a large network, the probability of taking a link formation decision at any time step t is independent on the global network structure G^t, and depends only on the current number of followees $\deg_i^+(t)$ of agent i.

Now that we know (from (B.8)) the probability that agent i forms a link at any given time t, we can derive the probability distribution of the EFT T_i for agent i as the distribution of the number of time steps needed for agent i to make $L^*_{\theta_i}(0)$ linking decisions. Let N_i^j for $j > 1$, be the number of time steps that agent i takes between forming links $j-1$ and j, and let N_i^1 be the number of time steps that agent i waits after its entrace date until it forms its first link. The EFT T_i can be written as

$$T_i = N_i^1 + \sum_{j=2}^{L^*_{\theta_i}(0)} N_i^j. \quad \text{(B.10)}$$

From the expressions in (B.8), it is easy to see that both N_i^1 and $N_i^j, j > 1$ can be interpreted as *geometric random*

$$\mathbb{P}(a_i^t = 1 \,|\, G^t) = \begin{cases} \frac{|\mathcal{V}_{\theta_i}^t|-1}{|\mathcal{V}^t|-1}, & \deg_i^+(t) = 0 \\ \frac{\gamma_{\theta_i} K_i^s(t)}{K_i(t)} \left(1 - \mathbb{P}(K_i(t)=0)\right) + (1-\gamma_{\theta_i} + \gamma_{\theta_i}\mathbb{P}(K_i(t)=0)) \frac{|\mathcal{V}_{\theta_i}^t|-N_i^s(t)-1}{|\mathcal{V}^t|-N_i^s(t)-N_i^d(t)-1}, & 0 < \deg_i^+(t) \le L_{\theta_i}^*(0) \\ 0, & \deg_i^+(t) > L_{\theta_i}^*(0) \end{cases} \tag{B.7}$$

$$\lim_{t\to\infty} \mathbb{P}\left(a_i^t = 1 \,\middle|\, G^t\right) = \begin{cases} p_{\theta_i}, & \deg_i^+(t) = 0 \\ \gamma_{\theta_i} + (1-\gamma_{\theta_i}) p_{\theta_i}, & 0 < \deg_i^+(t) \le L_{\theta_i}^*(0) \\ 0, & \deg_i^+(t) > L_{\theta_i}^*(0) \end{cases} \tag{B.8}$$

variable with a different success probabilities determined by the linking probabilities in (B.8) as follows

$$\begin{aligned} &N_i^1 \sim \text{Geom}(p_{\theta_i}), \\ &N_i^j \sim \text{Geom}(\gamma_{\theta_i} + (1-\gamma_{\theta_i})\, p_{\theta_i}), j = 2, \ldots, L_{\theta_i}^*(0), \end{aligned} \tag{B.11}$$

where N_i^1 is independent on $N_i^j, j > 1$, and all the variables $N_i^j, j > 1$ are i.i.d. The independence of the variables $N_i^j, j > 1$ follow from the fact that the linking probability in (B.8) is independent of the graph G^t for a large network, and hence the process $\{N_i^j\}_{j>1}$ is memoryless. Therefore, the distribution of EFT T_i for agent i in an asymptotically large network follows a distribution

$$f_{T_i}(T_i \,|\, \theta_i) = f_{N_i^1}(N_i^1) \star f_{N_i^2}(N_i^2) \star \ldots \star f_{N_i^{L_{\theta_i}^*(0)}}(N_i^{L_{\theta_i}^*(0)}),$$

where $\star$ is the convolution operator. Therefore, the distribution of the EFT for agent i converge to a steady-state distribution that is independent of the entry date i and is only dependent on the agent's type θ_i. From Scheffe's lemma, we know that convergence of the probability mass functions implies convergence in distribution, thus the sequence of EFTs $\{T_i\}_i$ converges in distribution for all types of agents. This concludes the proof of the first part of statement (2).

Now we compute the EEFT, which is simply given by $\overline{T}_i = \mathbb{E}[\sum_{j=1}^{L_{\theta_i}^*(0)} N_i^j]$, where

$$\mathbb{E}[N_i^j] = \begin{cases} \frac{1}{p_{\theta_i}}, & j = 1, \\ \frac{1}{(1-\gamma_{\theta_i})\, p_{\theta_i} + \gamma_{\theta_i}}, & 2 \le j \le L_{\theta_i}^*(0). \end{cases}$$

Therefore, the EEFT is given by

$$\begin{aligned} \overline{T}_i &= \mathbb{E}\left[N_i^1\right] + \sum_{j=2}^{L_{\theta_i}^*(0)} \mathbb{E}\left[N_i^j\right] \\ &= \frac{1}{p_{\theta_i}} + \frac{L_{\theta_i}^*(\alpha_{\theta_i}^s)}{(1-\gamma_{\theta_i})\, p_{\theta_i} + \gamma_{\theta_i}}, \end{aligned} \tag{B.12}$$

which concludes the proof of statement (2) of the Theorem.

APPENDIX C
PROOF OF COROLLARY 1

We first define the notion of *first-order stochastic dominance* as follows. A pdf (or pmf) $f(x)$ first-order stochastically dominates a pdf $g(x)$ if and only if $G(x) \ge F(x), \forall x$, with strict inequality for some values of x, where $F(x)$ and $G(x)$ are the cumulative density functions. In this proof, we will use *first-order stochastic dominance* and *stochastic dominance* interchangeably. For the two random variables x and y, if $f(x)$ stochastically dominates $f(y)$, then we say $y \succeq x$. In the following, we prove some useful Lemmas that will be utilized in proving this Theorem.

Lemma C.1. Let X_1, X_2, Y_1, and Y_2 be independent random variables, and let $Z_1 = X_1 + Y_1$ and $Z_2 = X_2 + Y_2$. If $X_1 \succeq X_2$ and $Y_1 \succeq Y_2$, then $Z_1 \succeq Z_2$.

Proof We prove the Lemma for continuous random variables; the result can be straightforwardly generalized to discrete random variables. Since $X_1 \succeq X_2$ and $Y_1 \succeq Y_2$, then we have $F_{X_1}(x_1) \le F_{X_2}(x_2)$, $F_{Y_1}(Y_1) \le F_{Y_2}(Y_2)$, $\int u(x_1) f(x_1) dx_1 \ge \int u(x_2) f(x_2) dx_2$, and $\int u(y_1) f(y_1) dy_1 \ge \int u(y_2) f(y_2) dy_2$, for any increasing function $u(.)$. Note that since $Z_1 = X_1 + Y_1$ and $Z_2 = X_2 + Y_2$, then we have that $F_{Z_1}(z_1) = \int F_{Y_1}(z_1 - x_1) f(x_1) dx_1$, and $F_{Z_2}(z_2) = \int F_{Y_2}(z_2 - x_2) f(x_2) dx_2$. Since $F_{Y_1}(Y_1) \le F_{Y_2}(Y_2)$ and $X_1 \succeq X_2$, then $F_{Z_1}(z_1) \le F_{Z_2}(z_2)$ and it follows that $Z_1 \succeq Z_2$. □

Lemma C.2. If $Z_1 = \sum_{i=1}^N X_i$ and $Z_2 = \sum_{i=1}^M X_i$, where $N > M$, and the variables $X_i, \forall i \le N$ are i.i.d non-negative random variables, then $Z_1 \succeq Z_2$.

Proof Let $\tilde{X}_1 = \sum_{i=1}^M X_i$, and $\tilde{X}_2 = \sum_{i=1}^{N-M} X_i$. We can write Z_1 as $Z_1 = \tilde{X}_1 + \tilde{X}_2$. The cdf of Z_1 is then given by $F_{Z_1}(z_1) = \int F_{X_1}(z_1 - \tilde{x}_2) f_{\tilde{x}_2}(\tilde{x}_2) d\tilde{x}_2$. Since $\int F_{X_1}(z_1 - \tilde{x}_2) f_{\tilde{x}_2}(\tilde{x}_2) d\tilde{x}_2 \le F_{X_1}(z_1), \forall z_1$, and since $F_{X_1}(z_1) = F_{Z_2}(z_1)$, then $F_{Z_1}(z) \le F_{Z_2}(z), \forall z$, and it follows that $Z_1 \succeq Z_2$. □

The statement of the Theorem entails a comparative statics analysis for the effect of 3 different exogenous parameters $(L_{\theta_i}^*(0), p_{\theta_i}, \gamma_{\theta_i})$ on the distribution of the EFT T_i (in terms of the FOSD) in an asymptotically large network. We have shown in Appendix B that the distribution of T_i converges to a steady-state distribution when the network is asymptotically large. Recall from (B.10) that $T_i = N_i^1 + \sum_{j=2}^{L_{\theta_i}^*(0)} N_i^j$. Define $\bar{N}_i^1 = \sum_{j=2}^{L_{\theta_i}^*(0)} N_i^j$, hence $T_i = N_i^1 + \bar{N}_i^1$. Since N_i^1 is a geometric random variable (see (B.11)), then the pmf of N_i^1 is given by

$$f_{N_i^1}(N_i^1 \,|\, \theta_i) = p_{\theta_i} (1 - p_{\theta_i})^{N_i^1 - 1}. \tag{C.1}$$

On the other hand, $\bar{N}_i^1$ is the same of $L_{\theta_i}^*(0) - 1$ i.i.d geometric random variable, and hence it follows a *negative*

binomial distribution with the following pmf:

$$f_{\bar{N}_i^1}(\bar{N}_i^1 \,|\, \theta_i) = \binom{\bar{N}_i^1 - 1}{L^*_{\theta_i}(0) - 2} p^{L^*_{\theta_i}(0)-1} \, (1-p)^{\bar{N}_i^1 - L^*_{\theta_i}(0)+1}, \tag{C.2}$$

where $p = (1-\gamma_{\theta_i})p_{\theta_i} + \gamma_{\theta_i}$. In the rest of this Section, we will use the results in (C.1) and (C.2), together with Lemmas C.1 and C.2 in order to verify the comparative statics in the Theorem statement. That is, we will show that if $h_k = 1, \forall k \in \Theta$, $\tilde{L}^*_{\theta_i}(0) \geq L^*_{\theta_i}(0)$, $\tilde{p}_{\theta_i} \geq p_{\theta_i}$, and $\tilde{\gamma}_{\theta_i} \geq \gamma_{\theta_i}$, then for an agent i entering an asymptotically large network we have that

$$\begin{aligned} &T_i\left(p_{\theta_i}, \gamma_{\theta_i}, L^*_{\theta_i}(0)\right) \succeq T_i\left(p_{\theta_i}, \tilde{\gamma}_{\theta_i}, L^*_{\theta_i}(0)\right), \\ &T_i\left(p_{\theta_i}, \gamma_{\theta_i}, L^*_{\theta_i}(0)\right) \succeq T_i\left(\tilde{p}_{\theta_i}, \gamma_{\theta_i}, L^*_{\theta_i}(0)\right), \\ &T_i\left(p_{\theta_i}, \gamma_{\theta_i}, L^*_{\theta_i}(0)\right) \preceq T_i\left(p_{\theta_i}, \gamma_{\theta_i}, \tilde{L}^*_{\theta_i}(0)\right), \end{aligned} \tag{C.3}$$

where $T_i\left(p_{\theta_i}, \gamma_{\theta_i}, L^*_{\theta_i}(0)\right)$ is the EFT associated with the exogenous parameter tuple $\left(p_{\theta_i}, \gamma_{\theta_i}, L^*_{\theta_i}(0)\right)$.

Recall that $T_i = N_i^1 + \bar{N}_i^1$. Based on (C.1) and (C.2), the cdf of the two random variables N_i^1 and $\bar{N}_i^1$ are given by

$$\begin{aligned} &F(N_i^1 \,|\, \theta_i) = 1 - (1-p_{\theta_i})^{N_i^1} \\ &F(\bar{N}_i^1 \,|\, \theta_i) = 1 - I_{1-p}(L^*_{\theta_i}(0), \bar{N}_i^1 - L^*_{\theta_i}(0) + 1), \end{aligned} \tag{C.4}$$

where $p = (1-\gamma_{\theta_i})p_{\theta_i} + \gamma_{\theta_i}$, $I_{1-p}(x,y)$ is the *regularized incomplete beta function*, which is defined in terms of the incomplete beta function $B(1-p; x, y) = \int_0^{1-p} z^{x-1}(1-z)^{y-1}dz$ as $I_{1-p}(x,y) = \frac{B(1-p;x,y)}{B(x,y)}$. The first derivative of $I_{1-p}(x,y)$ with respect to p is given by

$$\frac{\partial I_{1-p}(x,y)}{\partial p} = \frac{-(1-p)^{x-1}p^{y-1}}{B(x,y)} < 0,$$

thus, $I_{1-p}(x,y)$ is monotonically decreasing in p. Let $\tilde{p} = (1-\gamma_{\theta_i})\tilde{p}_{\theta_i} + \gamma_{\theta_i}$. If $\tilde{p}_{\theta_i} > p_{\theta_i}$, then $\tilde{p} > p$, and it follows that $1-(1-p)^{N_i^1} < 1-(1-\tilde{p})^{N_i^1}$, and $1 - I_{1-\tilde{p}}\left(L^*_{\theta_i}(0), \bar{N}_i^1 - L^*_{\theta_i}(0)+1\right) > 1 - I_{1-p}\left(L^*_{\theta_i}(0), \bar{N}_i^1 - L^*_{\theta_i}(0)+1\right)$, and hence we have that

$$\begin{aligned} F(N_i^1 \,|\, \tilde{p}_{\theta_i}) &\geq F(N_i^1 \,|\, p_{\theta_i}) \\ F(\bar{N}_i^1 \,|\, \tilde{p}_{\theta_i}) &\geq F(\bar{N}_i^1 \,|\, p_{\theta_i}). \end{aligned} \tag{C.5}$$

By combining the inequalities in (C.5) with Lemma C.1, it follows that $F(T_i \,|\, \tilde{p}_{\theta_i}) \geq F(T_i \,|\, p_{\theta_i})$ for all values of T_i. Therefore, we have that $T_i\left(p_{\theta_i}, h_{\theta_i}, \gamma_{\theta_i}, L^*_{\theta_i}(0)\right) \succeq T_i\left(\tilde{p}_{\theta_i}, h_{\theta_i}, \gamma_{\theta_i}, L^*_{\theta_i}(0)\right)$, which concludes the proof of the first comparative statics result in (C.3). The second comparative statics result in (C.3) can be proved in the exact same manner by defining $\tilde{p}$ as $\tilde{p} = ((1-\tilde{\gamma}_{\theta_i})p_{\theta_i} + \tilde{\gamma}_{\theta_i})$ and then following the same procedure.

Finally, since $T_i = N_i^1 + \sum_{j=2}^{L^*_{\theta_i}(0)-1} N_i^j$, then it follows directly from Lemma C.2 that if $\tilde{L}^*_{\theta_i}(0) > L^*_{\theta_i}(0)$, then $T_i(p_{\theta_i}, h_{\theta_i}, \gamma_{\theta_i}, \tilde{L}^*_{\theta_i}(0)) \succeq T_i(p_{\theta_i}, h_{\theta_i}, \gamma_{\theta_i}, L^*_{\theta_i}(0))$.

APPENDIX D
PROOF OF THEOREM 2

Since $\alpha_\theta^s \geq \alpha_\theta^d, \forall \theta \in \Theta$, the utility function of an agent i is maximized when this agent is linked $L^*_{\theta_i}(0)$ similar-type agents; the (saturated) utility function achieved by such an agent is $u_i^t = v_{\theta_i}(\alpha^s_{\theta_i} L^*_{\theta_i}(0)) - c\, L^*_{\theta_i}(0)$ at time t. Consequently, the average utility of agents in the network at time t, $U^t = \frac{1}{t}\sum_i u_i^t$, is maximized when every agent i in the network achieves a utility of $u_i^t = v_{\theta_i}(\alpha^s_{\theta_i} L^*_{\theta_i}(0)) - c\, L^*_{\theta_i}(0)$. Thus, the optimal average network utility $\overline{U}^*$ at time t is

$$\overline{U}^* = \sum_{\theta \in \Theta} \frac{|\mathcal{V}_\theta^t|}{t}(v_\theta(\alpha_\theta^s L^*_\theta(0)) - c\, L^*_\theta(0)),$$

which for a large network converges to

$$\overline{U}^* = \sum_{\theta \in \Theta} p_\theta \left(v_\theta\left(\alpha_\theta^s L^*_\theta(0)\right) - c\, L^*_\theta(0)\right). \tag{D.1}$$

We want to prove the following statement

$$\left(\lim_{t\to\infty} U^t = \overline{U}^*\right) \iff (h_\theta = 1, \forall \theta \in \Theta). \tag{D.2}$$

We prove the statement in (D.2) by proving the following sufficiency and necessity statements:

$$\begin{aligned} &\left(\lim_{t\to\infty} U^t = \overline{U}^*\right) \Rightarrow (h_\theta = 1, \forall \theta \in \Theta). \\ &\left(\lim_{t\to\infty} U^t = \overline{U}^*\right) \Leftarrow (h_\theta = 1, \forall \theta \in \Theta). \end{aligned} \tag{D.3}$$

We start by proving the sufficiency condition by contradiction. Assume that $\lim_{t\to\infty} U^t = \overline{U}^*$ and that there exists exactly one type of agents $\theta \in \Theta$ with $h_\theta < 1$. When $h_\theta < 1$, a type-θ agent i will form a link with a different-type agent upon its entrance in the network if $\theta_{m_i(i)} \neq \theta$. In a large network, there exists a fraction of $(1-p_\theta)$ type-θ agents that would have met a different-type agent upon its entrance in the network. Since $\alpha_\theta^s \geq \alpha_\theta^d$, the utility achieved by those agents is strictly less than $u_i^t = v_{\theta_i}(\alpha^s_{\theta_i} L^*_{\theta_i}(0)) - c\, L^*_{\theta_i}(0)$, and hence it follows that $\lim_{t\to\infty} U^t < \overline{U}^*$, which contradicts with the fact that $\lim_{t\to\infty} U^t = \overline{U}^*$. Therefore, if $\lim_{t\to\infty} U^t = \overline{U}^*$, then all agents must be extremely homophilic.

Now we prove the converse. If all agents are extremely homophilic, then each agent connects only to similar type agents, i.e. $\mathbb{P}\left(a_i^t \,|\, \theta_{m_i(t)}\right) = \mathbf{1}_{\left\{\theta_{m_i(t)} = \theta_i, N_i(t) < L^*_{\theta_i}(0)\right\}}$. Since each agent meets a same-type agent with a non-zero probability in every time step, and will always form its ego network in a finite time (recall Lemma 1), the utility achieved by each agent i is then given by $v_{\theta_i}\left(\alpha^s_{\theta_i} L^*_{\theta_i}(0)\right) - c\, L^*_{\theta_i}(0)$, and it follows that $\lim_{t\to\infty} U^t = \overline{U}^*$.

Finally, since when $h_\theta = 1, \forall \theta \in \Theta$, agents restrict their links to same-type agents only, then there is no links between different groups and the network will be disconnected with the number of components being at least equal to the number of types, i.e. $\omega\left(G^t\right) \geq |\Theta|$.

APPENDIX E
PROOF OF THEOREM 3

The first and second statements of the Theorem quantify the popularity growth in non-opportunistic and opportunistic societies, respectively. We prove the first statement in Subsection E.1, and then prove the second statement in Subsection E.2.

E.1 Popularity Growth in Non-opportunistic Societies

We start by evaluating the popularity growth rate of a given agent i in a tolerant society with fully non-opportunistic agents, i.e. a society with $h_k = 0, \gamma_k = 0, \forall k \in \Theta$. Note that the popularity of agent i at time $t \geq i$ can be written as $\deg_i^-(t) = \sum_{j=i}^{t} \Delta\deg_i^-(j)$, and hence the expected popularity of agent i is given by

$$\mathbb{E}[\deg_i^-(t)] = \mathbb{E}\left[\sum_{j=i}^{t} \Delta\deg_i^-(j)\right] = \sum_{j=i}^{t} \mathbb{E}[\Delta\deg_i^-(j)], \tag{E.1}$$

where the expectation is taken over all the realizations of the graph process $\{G^t\}_{t=1}^{\infty}$; therefore, by the *the law of total expectation* we have that

$$\mathbb{E}[\Delta\deg_i^-(t)] = \mathbb{E}\left[\mathbb{E}[\Delta\deg_i^-(t) \,|\, G^t]\right] = \sum_{G^t \in \mathcal{G}^t} \mathbb{E}[\Delta\deg_i^-(t) \,|\, G^t] \cdot \mathbb{P}(G^t). \tag{E.2}$$

Recall that from Theorem 1, we know that in a tolerant society ($h_k = 0, \forall k \in \Theta$), each agent j stays $L^*_{\theta_j}(0)$ time steps in the ego network formation process, i.e. $T_j = L^*_{\theta_j}(0)$ with probability 1. Thus, at any given time t, the set of agents that can potentially link to agent i, which we denote as Φ^t, in any realization of G^t is given by

$$\Phi^t = \{t - L^* + 1, t - L^* + 2, \ldots, t\},$$

where $L^* = \max_{\theta \in \Theta} L^*_\theta(0)$. The set Φ^t comprises the most recent L^* agents entering the network; since only these agents can link to agent i at time t, the subgraph G^t_Φ induced by the set of agents in Φ^t is a sufficient statistic for $\Delta\deg_i^-(t)$. That is, we have that

$$\Delta\deg_i^-(t) \,|\, G^t \overset{d}{=} \Delta\deg_i^-(t) \,|\, G^t_\Phi, \tag{E.3}$$

where $\overset{d}{=}$ denotes equality in distribution. The condition in (E.3) means that the distribution of the number of links acquired by an agent at any given time depends only on what happened in the network in the previous L^* time steps, which is a direct consequence of Theorem 1(a). Based on (E.3), (E.2) can be written as

$$\mathbb{E}[\Delta\deg_i^-(t)] = \sum_{G^t_\Phi} \mathbb{E}[\Delta\deg_i^-(t) \,|\, G^t_\Phi] \cdot \mathbb{P}(G^t_\Phi). \tag{E.4}$$

The expectation in (E.4) is analytically evaluated for an asymptotically large network through the steps (a)-(h) in (E.5). In the following, we explain the steps involved in (E.5). In step (a), we start by the result in (E.4) in which we apply the law of total expectation by marginalizing the conditional expectation of the random variable $\Delta\deg_i^-(t) \,|\, G^t_\Phi$ over the distribution of the subgraph G^t_Φ. The random variable $\Delta\deg_i^-(t) \,|\, G^t_\Phi$ corresponds to the number of links gained by agent i at time t; since there are only L^* agents that can form links at time t (i.e. the members of the set Φ^t), then the distribution of $\Delta\deg_i^-(t) \,|\, G^t_\Phi$ has a support $\{0, 1, \ldots, L^*\}$. Since $h_\theta = 0, \forall \theta \in \Theta$, then every agent that meets agent i at time t will link to it, and hence we have that

$$\Delta\deg_i^-(t) \,|\, G^t_\Phi = \sum_{k \in \Phi^t} \mathbf{1}_{\{m_k(t) = i\}} \,|\, G^t_\Phi.$$

Step (b) follows by substituting the equation above in (a), and realizing that $\mathbb{E}[\mathbf{1}_{\{m_k(t)=i\}} \,|\, G^t_\Phi] = \mathbb{P}(m_k(t) = i \,|\, G^t_\Phi)$. Note that since $\gamma_\theta = 0, \forall \theta \in \Theta$, then the meeting process is independent of the network structure G^t_Φ (refer to Section 3.3), i.e. the probability that agent $k \in \Phi^t$ meets agent i at time t is given by

$$\mathbb{P}(m_k(t) = i \,|\, G^t_\Phi) = \frac{\mathbf{1}\left\{k \notin \mathcal{N}^-_{i,t-1}, L^*_{\theta_k}(0) \geq t - k + 1\right\}}{t - 1 - |\mathcal{N}^+_{k,t}|}. \tag{E.6}$$

That is, if $k \in \mathcal{N}^-_{i,t-1}$ (k is already linked to i) or if $L^*_{\theta_k}(0) < t - k + 1$ (k has already formed $L^*_{\theta_k}(0)$ links and saturated its utility function), then k meets i with probability 0, otherwise, k meets agents picked randomly from the set of agents that are not in its followee set $\mathcal{N}^+_{k,t}$, and hence it meets i with probability $\frac{1}{t-1-|\mathcal{N}^+_{k,t}|}$. From the above expression of $\mathbb{P}(m_k(t) = i \,|\, G^t_\Phi)$, we observe that the θ_k and the events $k \in \mathcal{N}^-_{i,t-1}$ and $L^*_{\theta_k}(0) < t - k + 1$ are sufficient statistics for the event $(m_k(t) = i)$, and hence we have that

$$\mathbb{P}(m_k(t) = i \,|\, G^t_\Phi) = \mathbb{P}(m_k(t) = i \,|\, k \notin \mathcal{N}^-_{i,t-1}, L^*_{\theta_k}(0) \geq t - k + 1, \theta_k).$$

Step (c) substitutes the above in the result of step (b). In step (d), we plug in the expression in (E.6) into the result of step (c) after observing that $|\mathcal{N}^+_{k,t}| = t - k$, which follows from the fact that, since $h_k = 0$, agent k forms exactly one link at each time step since its entrance. In step (d), the probability $\mathbb{P}(k \notin \mathcal{N}^-_{i,t-1}, L^*_{\theta_k}(0) \geq t - k + 1 \,|\, \theta_k)$ is evaluated by noting that for $L^*_{\theta_k}(0) \geq t - k + 1$, we have that

$$\mathbb{P}(k \notin \mathcal{N}^-_{i,t-1}, L^*_{\theta_k}(0) \geq t - k + 1 \,|\, \theta_k) = \prod_{n=k}^{t-1} \mathbb{P}(m_k(n) = i \,|\, k \notin \mathcal{N}^-_{i,n-1}, \theta_k).$$

That is, the probability that k is not linked to i by time t is the equal to the probability that k has never met i in the previous $t - k$ time steps. Step (e) is a straightforward algebraic simplification of (d). A standard Taylor series approximation $(1 - \frac{1}{x})^a \approx e^{-\frac{a}{x}}$ is used in (f)-(g), and the asymptotic value of $\mathbb{E}[\deg_i^-(t)]$ for a large t is provided in (h).

The expected popularity of agent i at time $t \geq i$ can thus be characterized by combining (E.1) and (E.5) as follows

$$\begin{aligned}
\mathbb{E}[\deg_i^-(t)] &= \sum_{j=i}^{t} \mathbb{E}[\Delta\deg_i^-(j)] \\
&= \sum_{j=i}^{t} \frac{\bar{L}}{j} \\
&= \bar{L}\,(H_t - H_{i-1}) \\
&\asymp \bar{L}\,((\log(t) - \psi) - (\log(i-1) - \psi)) \\
&= \bar{L} \log\left(\frac{t}{i-1}\right),
\end{aligned} \tag{E.7}$$

where H_N is the N^{th} harmonic number, and ψ is the *Euler-Mascheroni* constant. If follows from (E.7) that $\mathbb{E}[\deg_i^-(t)]$ is $O(\bar{L}\log(t))$, which concludes the proof of the first statement of the Theorem.

$$
\begin{aligned}
\mathbb{E}[\Delta \deg_i^-(t)] &\overset{(a)}{=} \sum_{G_\Phi^t} \mathbb{E}[\Delta \deg_i^-(t) \,|\, G_\Phi^t] \cdot \mathbb{P}(G_\Phi^t) \\
&\overset{(b)}{=} \sum_{G_\Phi^t} \sum_{k \in \Phi^t} \mathbb{E}[\mathbf{1}_{\{m_k(t)=i\}} \,|\, G_\Phi^t] \cdot \mathbb{P}(G_\Phi^t) = \sum_{G_\Phi^t} \sum_{k \in \Phi^t} \mathbb{P}(m_k(t)=i \,|\, G_\Phi^t) \cdot \mathbb{P}(G_\Phi^t) \\
&\overset{(c)}{=} \sum_{k \in \Phi^t} \sum_{\theta_k \in \Theta} p_{\theta_k} \, \mathbb{P}(m_k(t)=i \,|\, k \notin \mathcal{N}_{i,t-1}^-, L_{\theta_k}^*(0) \geq t-k+1, \theta_k) \, \mathbb{P}(k \notin \mathcal{N}_{i,t-1}^-, L_{\theta_k}^*(0) \geq t-k+1 \,|\, \theta_k) \\
&\overset{(d)}{=} \sum_{k \in \Phi^t} \sum_{\theta_k \in \Theta} p_{\theta_k} \cdot \mathbf{1}_{\left\{L_{\theta_k}^*(0) \geq t-k+1\right\}} \cdot \left(\frac{1}{t-1-(t-k)}\right) \prod_{n=k}^{t-1} \left(1 - \frac{1}{n-1-(n-k)}\right) \\
&\overset{(e)}{=} \sum_{k \in \Phi^t} \sum_{\theta_k \in \Theta} p_{\theta_k} \cdot \mathbf{1}_{\left\{L_{\theta_k}^*(0) \geq t-k+1\right\}} \cdot \left(\frac{1}{k-1}\right) \left(1 - \frac{1}{k-1}\right)^{t-k} \\
&\overset{(f)}{\approx} \sum_{k=t-L^*+1}^{t} \sum_{\theta_k \in \Theta} p_{\theta_k} \cdot \mathbf{1}_{\left\{L_{\theta_k}^*(0) \geq t-k+1\right\}} \cdot \left(\frac{e^{-\frac{t-k}{k-1}}}{k-1}\right) \\
&\overset{(g)}{=} \sum_{w=1}^{L^*} \sum_{\theta \in \Theta} p_\theta \mathbf{1}_{\left\{L_\theta^*(0) \geq w\right\}} \left(\frac{e^{-\frac{w-1}{t-w}}}{t-w}\right) \asymp \frac{1}{t} \underbrace{\sum_{w=1}^{L^*} \sum_{\theta \in \Theta} p_\theta \mathbf{1}_{\left\{L_\theta^*(0) \geq w\right\}}}_{\bar{L}} \overset{(h)}{=} \frac{\bar{L}}{t}
\end{aligned}
\tag{E.5}
$$

E.2 Popularity Growth in Opportunistic Societies

Now we evaluate the popularity growth rate in a tolerant society with fully opportunistic agents, i.e. a society with $h_k = 0, \gamma_k = 1, \forall k \in \Theta$. Following the same steps as in Subsection E.1, we start by evaluating the expected number of links gained by agent i, $\mathbb{E}[\Delta \deg_i^-(t)]$, using the law of iterated expectation $\mathbb{E}[\Delta \deg_i^-(t)] = \mathbb{E}\left[\mathbb{E}[\Delta \deg_i^-(t) \,|\, G_\Phi^t]\right]$ in (E.6), where G_Φ^t is the subgraph induced by the length-2 ego networks of agents in Φ. (Unlike the case in Subsection E.1, here the sufficient statistic for $\Delta \deg_i^-(t)$ is the length-2 ego networks of agents in Φ, since the agents in Φ are fully opportunistic and can only meet the followees of their followees.)

In what follows, we explain steps (a)-(g) involved in the derivation in (E.6). Steps (a)-(b) are identical to steps (a)-(b) in (E.5). The probability that agent k meets agent i at time t is given by

$$
\mathbb{P}(m_k(t)=i \,|\, G_\Phi^t) = \begin{cases} \frac{1}{t-1}, & k=t, \\ \frac{\mathbf{1}_{\left\{\mathbb{F}_1(k,i), \mathbb{F}_2(k,i), L_{\theta_k}^*(0) \geq t-k+1\right\}}}{K_k(t)}, & k \neq t, \end{cases}
\tag{E.9}
$$

where

$$
\begin{aligned}
\mathbb{F}_1(k,i) &:= k \notin \mathcal{N}_{i,t-1}^- \\
\mathbb{F}_2(k,i) &:= k \in \cup_{j \in \mathcal{N}_{i,t-1}^-} \mathcal{N}_{j,t-1}^-.
\end{aligned}
$$

The expression in (E.9) follows directly from the meeting process in Section 3.3. Agent $k=t$ has no followees since it has just entered the network, and hence it meets agents picked randomly from G^t, thus we have that $\mathbb{P}(m_t(t) = i \,|\, G_\Phi^t) = \frac{1}{t-1}$. For any $k < t$, agent k meets agent i only if the two agents have not met in a previous time step (event $\mathbb{F}_1(k,i)$), and i resides in agent k's set of followees of followees (event $\mathbb{F}_2(k,i)$); such a meeting happens with a probability $\frac{1}{K_k(t)}$. In step (c), we substitute (E.9) in the result of step (b). In step (d), we pull out the term corresponding to $k = t-1$ in the summation in (c). Step (e) follows by substituting the following expressions in (d):

$$
\begin{aligned}
\mathbb{P}(\mathbb{F}_1(t-1,i) \,|\, G_\Phi^t) &= (1 - \frac{1}{t-2}), \\
\mathbb{P}(\mathbb{F}_2(t-1,i) \,|\, G_\Phi^t) &= \frac{\deg_i^-(t-1)}{t-2}.
\end{aligned}
$$

That is, the probability that agent $t-1$ is not a followee of i at time t is equal to the probability that $t-1$ has not met i upon its entrance, which is given by $(1 - \frac{1}{t-2})$. Furthermore, the probability that agent i is a followee of a followee of $t-1$ at time t is equal to the probability that agent $t-1$ has linked to one of the followers of i at time step $t-1$, which is given by $\frac{\deg_i^-(t-1)}{t-2}$. Since type-$m$ agents form $L_m^*(0)$ links, then we have that $K_{t-1}(t) = L_{m_{t-1}(t-1)}^*(0)$. The rest of the derivation follows straightforwardly.

In order to find a lower bound on the growth rate of $\mathbb{E}[\deg_i^-(t)]$, we analyze a process that is described by

$$
\mathbb{E}[\Delta \deg_i^-(t)] = \frac{1}{t} \left(1 + b \, \mathbb{E}[\deg_i^-(t-1)]\right),
\tag{E.10}
$$

where $b = \sum_{w \in \Theta} p_w / L_w^*(0)$. Following [29] and [24], we adopt a continuous-time mean-field approximation for the popularity growth process. That is, since $\mathbb{E}[\Delta \deg_i^-(t)] = \mathbb{E}[\deg_i^-(t)] - \mathbb{E}[\deg_i^-(t-1)]$, we adopt the following approximation

$$
\frac{\partial \mathbb{E}[\deg_i^-(t)]}{\partial t} \approx \mathbb{E}[\Delta \deg_i^-(t)].
\tag{E.11}
$$

Combining (E.10) and (E.11), the popularity of each agent i is governed by the following differential equation

$$
\frac{\partial \mathbb{E}[\deg_i^-(t)]}{\partial t} = \frac{1}{t} \left(1 + b \, \mathbb{E}[\deg_i^-(t)]\right).
\tag{E.12}
$$

$$
\begin{aligned}
\mathbb{E}[\Delta\deg_i^-(t)] &\overset{(a)}{=} \sum_{G_\Phi^t} \mathbb{E}[\Delta\deg_i^-(t) \,|\, G_\Phi^t] \cdot \mathbb{P}(G_\Phi^t) \\
&\overset{(b)}{=} \sum_{G_\Phi^t} \sum_{k\in\Phi^t} \mathbb{P}(m_k(t)=i \,|\, G_\Phi^t) \cdot \mathbb{P}(G_\Phi^t) \\
&\overset{(c)}{=} \frac{1}{t-1} + \sum_{G_\Phi^t} \sum_{k\in\Phi^t/\{t\}} \frac{1}{K_k(t)} \cdot \mathbf{1}_{\left\{\mathbb{F}_1(k,i),\mathbb{F}_2(k,i),L^*_{\theta_k}(0)\geq t-k+1\right\}} \cdot \mathbb{P}(G_\Phi^t) \\
&\overset{(d)}{=} \frac{1}{t-1} + \sum_{G_\Phi^t} \frac{1}{L^*_{\theta_{m_{t-1}(t-1)}}(0)} \cdot \mathbf{1}_{\left\{\mathbb{F}_1(t-1,i),\mathbb{F}_2(t-1,i),L^*_{\theta_{t-1}}(0)\geq 2\right\}} \cdot \mathbb{P}(G_\Phi^t) \\
&\quad + \underbrace{\sum_{G_\Phi^t} \sum_{k\in\Phi^t/\{t,t-1\}} \frac{1}{K_k(t)} \cdot \mathbf{1}_{\left\{\mathbb{F}_1(k,i),\mathbb{F}_2(k,i),L^*_{\theta_k}(0)\geq t-k+1\right\}} \cdot \mathbb{P}(G_\Phi^t)}_{\Psi\geq 0} \\
&\overset{(e)}{=} \frac{1}{t-1} + \mathbb{E}\left[\frac{\deg_i^-(t-1)}{t-2}\right] \cdot \left(1-\frac{1}{t-2}\right) \cdot \left(\sum_{\theta_{t-1}\in\Theta} p_{\theta_{t-1}} \mathbf{1}_{\left\{L^*_{\theta_{t-1}}(0)\geq 2\right\}}\right) \sum_{m\in\Theta} \frac{p_m}{L^*_m(0)} + \Psi \\
&\overset{(f)}{\geq} \frac{1}{t-1} + \mathbb{E}\left[\frac{\deg_i^-(t-1)}{t-2}\right] \cdot \left(1-\frac{1}{t-2}\right) \cdot \sum_{m\in\Theta} \frac{p_m}{L^*_m(0)} \\
&\overset{(g)}{\approx} \frac{1}{t} + \frac{1}{t}\mathbb{E}\left[\deg_i^-(t-1)\right] \cdot \sum_{w\in\Theta} \frac{p_w}{L^*_w(0)}.
\end{aligned} \tag{E.6}
$$

The differential equation in (E.12) can be solved by dividing both sides by $(1 + b\,\mathbb{E}[\deg_i^-(t)])$ and then integrating both sides as follows

$$\int \frac{1}{(1+b\,\mathbb{E}[\deg_i^-(t)])}\, d\mathbb{E}[\deg_i^-(t)] = \int \frac{1}{t}\, dt,$$

which reduces to

$$\frac{1}{b}\log(1+b\,\mathbb{E}[\deg_i^-(t)]) + c_1 = \log(t) + c_2,$$

and hence we have that

$$\mathbb{E}[\deg_i^-(t)] = c_3 t^b - \frac{1}{b},$$

where c_1, c_2, and c_3 are constants. The constant c_3 can be obtained from the initial conditions as follows. Note that at $t = i$, agent i cannot receive any links since all agents in the network are opportunistic, and the set of followers of i is empty at its entry date. Thus, $\deg_i^-(t) = 0$, $\mathbb{E}[\deg_i^-(t)] = 0$, which means that $c_3 = \frac{1}{i^b b}$, and it follows that

$$\mathbb{E}[\deg_i^-(t)] \geq \frac{1}{b}\left(\frac{t}{i}\right)^b - \frac{1}{b}, \forall t \geq i,$$

which implies that the popularity of every agent i grows at least sublinearly in time. This concludes the proof of the second statement of the Theorem.

APPENDIX F
PROOF OF COROLLARY 3

Recall that from Definition 1, we say that preferential attachment emerges if

$$\deg_i^-(t) \geq \deg_j^-(t) \Rightarrow \Delta\deg_i^-(t) \succeq \Delta\deg_j^-(t),$$

for all $i, j \leq t, t \in \mathbb{N}$. From Appendix E, we know that in a tolerant society ($h_\theta = 0, \forall\theta \in \Theta$), the set of agents that can potentially link to a given agent i at time step t in any realization of G^t is

$$\Phi^t = \{t - L^* + 1, t - L^* + 2, \ldots, t\},$$

where $L^* = \max_{\theta\in\Theta} L^*_\theta(0)$. Let $p^t_{ik}(d)$ be the probability that agent $k \in \Phi^t$ links to agent i at time t given that $\deg_i^-(t) = d$; from Appendix F, we know that $p^t_{ik}(d)$ is given by

$$p^t_{ik}(d) = \mathbb{P}(m_k(t) = i \,|\, \deg_i^-(t) = d).$$

From Subsection E.2, we know that $p^t_{ik}(d) \geq p^t_{ik}(d')$ for $d \geq d'$. The random variable $\Delta\deg_i^-(t) \,|\, \deg_i^-(t) = d$ is equivalent to $\sum_{k\in\Phi^t} \mathbf{1}_{\{m_k(t)=i\}} \,|\, \deg_i^-(t) = d$, and hence it obeys a *Poisson binomial distribution* with a support $\{0, 1, \ldots, L^*\}$. Therefore, the pmf of $\Delta\deg_i^-(t) \,|\, \deg_i^-(t) = d$ is given by

$$\mathbb{P}(\Delta\deg_i^-(t) = n \,|\, \deg_i^-(t) = d) = \sum_{A\in S_n^t} \prod_{q\in A} p^t_{iq}(d) \prod_{r\in A^c} (1 - p^t_{ir}(d)),$$

where S_n^t is the set of all size-n subsets of Φ^t. The CDF of $\Delta\deg_i^-(t) \,|\, \deg_i^-(t) = d$ is given by

$$\mathbb{P}(\Delta\deg_i^-(t) \leq n \,|\, \deg_i^-(t) = d) = \sum_{l=0}^{n} \sum_{A\in S_l^t} \prod_{q\in A} p^t_{iq} \prod_{r\in A^c} (1 - p^t_{ir}).$$

It can be easily shown that

$$\frac{\partial\mathbb{P}(\Delta\deg_i^-(t) \leq n \,|\, \deg_i^-(t) = d)}{\partial p^t_{ik}(d)} < 0,$$

$\forall k \in \Phi^t$. Thus, if $d \geq d'$, then $p^t_{ik}(d) \geq p^t_{ik}(d')$ and hence

$$\mathbb{P}(\Delta\deg_i^-(t) \leq n \,|\, \deg_i^-(t) = d) \leq \mathbb{P}(\Delta\deg_i^-(t) \leq n \,|\, \deg_i^-(t) = d'),$$

$\forall n$, which implies that

$$\Delta\deg_i^-(t) \leq n \,|\, \deg_i^-(t) = d \succeq \Delta\deg_j^-(t) \leq n \,|\, \deg_i^-(t) = d'.$$

This conclude the proof of the Corollary.

APPENDIX G
PROOF OF COROLLARY 4

Note that for agents i and j, $i < j$, we can write $\deg_i^-(t)$ as

$$\deg_i^-(t) = \sum_{m=i}^{j-1} \Delta\deg_i^-(m) + \sum_{n=j}^{t} \Delta\deg_i^-(n),$$

whereas $\deg_j^-(t)$ can be written as

$$\deg_j^-(t) = \sum_{n=j}^{t} \Delta\deg_j^-(n).$$

Since $\sum_{n=j}^{t} \Delta\deg_j^-(n) \overset{d}{=} \sum_{n=j}^{t} \Delta\deg_i^-(n)$, then it follows from the result of Lemma C.1 that $\deg_i^-(t) \succeq \deg_j^-(t)$.

APPENDIX H
PROOF OF COROLLARY 5

From (E.7), we know that

$$\mathbb{E}[d_i^1(t)] = \bar{L} \log\left(\frac{t}{i-1}\right),$$

whereas $\mathbb{E}[d_i^2(t)]$ is lower-bounded as follows (refer to Subsection E.1)

$$\mathbb{E}[d_i^2(t)] \geq \frac{1}{b}\left(\left(\frac{t}{i}\right)^b - 1\right).$$

Since $\mathbb{E}[d_i^2(t)]$ grows faster than $\mathbb{E}[d_i^1(t)]$, we know that $\mathbb{E}[d_i^2(t)]$ dominates $\mathbb{E}[d_i^1(t)]$ after a finite time T^*. We can obtain T^* by solving the following transcendental equation for t

$$\mathbb{E}[d_i^1(t)] = \mathbb{E}[d_i^2(t)].$$

An upper-bound on the solution can be obtained by solving the following transcendental equation for t, in which we substitute for $\mathbb{E}[d_i^2(t)]$ by its lower-bound

$$\bar{L} \log\left(\frac{t}{i-1}\right) = \frac{1}{b}\left(\left(\frac{t}{i}\right)^b - 1\right). \tag{H.1}$$

Through straightforward algebraic manipulations, (H.1) can be put in the following form

$$t^{b\bar{L}} = \frac{(i-1)^{b\bar{L}}}{e}\, e^{\left(\frac{t}{i}\right)^b}. \tag{H.2}$$

A functional form for the solution to (H.2) can be obtained in terms of the Lambert W function $\mathcal{W}_{-1}(.)$ [56] as follows

$$t^* = i \times \left(-\bar{L}\mathcal{W}_{-1}\left(\frac{-1}{\bar{L}} e^{\frac{-1}{\bar{L}}}\right)\right)^{\frac{1}{b}}, \tag{H.3}$$

which concludes the proof of the Theorem.

APPENDIX I
PROOF OF THEOREM 4

The first and second statements of the Theorem quantify the popularity growth in non-opportunistic and opportunistic societies, respectively, where in both cases we study societies that are extremely homophilic. The proof of the Theorem follows the same steps involved in the proof of Theorem 3 (Appendix E). In what follows, we prove the first statement of the Theorem; the proof of the second statement parallels exactly the proof in Subsection E.2.

In (I.3), we follow the same analysis approach used in (E.5) in Appendix E; we derive an expression for the expected number of links gained by an agent i at time t, i.e. $\mathbb{E}[\Delta\deg_i^-(t) \,|\, \theta_i]$, using the law of total expectation. Steps (a) and (b) are similar to setps (a) and (b) in (E.5), with the following two differences: (1) since agents are extremely homophilic, then it does not hold that each agent k has an EFT of $L_k^*(0)$ (Theorem 1), and hence G_Φ^t is not a sufficient statistic for the random variable $\Delta\deg_i^-(t) \,|\, \theta_i$ as in (E.5), and (2) agents link to each other when they meet only if they are of the same type, i.e. agent k links to i at time t if $m_k(t) = i$ and $\theta_k = \theta_i$. Steps (c) and (d) follow by observing that $\mathbb{P}(m_k(t) = i, \theta_k = \theta_i \,|\, G^t) = p_{\theta_i}\mathbb{P}(m_k(t) = i, \,|\, G^t, \theta_k = \theta_i)$, and

$$\begin{aligned}\mathbb{P}(m_k(t) = i, \,|\, G^t, \theta_k = \theta_i) &= \frac{\mathbf{1}_{\left\{k \notin \mathcal{N}_{i,t-1}^-, \deg_k^+(t) < L_{\theta_k}^*(0)\right\}}}{t - \deg_k^+(t) - 1} \\ &\approx \frac{1}{t}\,\mathbf{1}_{\left\{k \notin \mathcal{N}_{i,t-1}^-, \deg_k^+(t) < L_{\theta_k}^*(0)\right\}}.\end{aligned}$$

That is, since $\gamma_k = 0$ (meetings are random), k meets i with probability $\frac{1}{t}$ (in a large enough network) if i is not already linked to k ($k \notin \mathcal{N}_{i,t-1}^-$), and k has not yet formed its ego network ($\deg_k^+(t) < L_{\theta_k}^*(0)$); k meets i with probability 0 otherwise. Thus, the event $m_k(t) = i$ depends only on the step graph G^t only through the conditions ($k \notin \mathcal{N}_{i,t-1}^-$) and ($\deg_k^+(t) < L_{\theta_k}^*(0)$); for an asymptotically large network, $\mathbb{P}(k \notin \mathcal{N}_{i,t-1}^- \,|\, \theta_i) \to 1$, which leads to the approximation in step (d). The term $\sum_{k=1}^{t} \mathbb{P}(\deg_k^+(t) < L_{\theta_k}^*(0) \,|\, \theta_k = \theta_i)$ corresponds to the expected number of agents who have not formed their ego networks by time t. In step (e), we replace this term with its mean-field approximation, i.e. we assume that each agent has a deterministic EFT that is equal to its expected EFT, which we know from Theorem 1 that it is given by $\mathbb{E}[T_k \,|\, \theta_k] = \frac{L_{\theta_k}^*(0)}{p_{\theta_k}}$. This leads to the expression in (f).

Based on (I.3), the popularity of an agent i at time t is given by

$$\begin{aligned}\mathbb{E}[\deg_i^-(t) \,|\, \theta_i] &= \sum_{j=i}^{t} \mathbb{E}[\Delta\deg_i^-(j) \,|\, \theta_i] \\ &= \sum_{j=i}^{t} \frac{L_{\theta_i}^*(0)}{j} \\ &= L_{\theta_i}^*(0)(H_t - H_{i-1}) \\ &\asymp L_{\theta_i}^*(0) \log\left(\frac{t}{i-1}\right),\end{aligned} \tag{I.2}$$

which concludes the proof of the Theorem.

$$
\begin{aligned}
\mathbb{E}[\Delta \deg_i^-(t) \,|\, \theta_i] &\overset{(a)}{=} \sum_{G^t} \mathbb{E}[\Delta \deg_i^-(t) \,|\, G^t, \theta_i] \cdot \mathbb{P}(G^t \,|\, \theta_i) \\
&\overset{(b)}{=} \sum_{G^t} \sum_{k=1}^{t} \mathbb{E}[\mathbf{1}_{\{m_k(t)=i, \theta_k=\theta_i\}} \,|\, G^t, \theta_i] \cdot \mathbb{P}(G^t \,|\, \theta_i) = \sum_{G^t} \sum_{k=1}^{t} \mathbb{P}(m_k(t)=i, \theta_k=\theta_i \,|\, G^t, \theta_i) \cdot \mathbb{P}(G^t \,|\, \theta_i) \\
&\overset{(c)}{\approx} \sum_{k=1}^{t} p_{\theta_i} \mathbb{P}(m_k(t)=i \,|\, k \notin \mathcal{N}_{i,t-1}^-, \deg_k^+(t) < L_{\theta_k}^*(0), \theta_k=\theta_i) \mathbb{P}(k \notin \mathcal{N}_{i,t-1}^-, \deg_k^+(t) < L_{\theta_k}^*(0) \,|\, \theta_k=\theta_i) \\
&\overset{(d)}{=} \frac{p_{\theta_i}}{t} \sum_{k=1}^{t} \mathbb{P}(k \notin \mathcal{N}_{i,t-1}^-, \deg_k^+(t) < L_{\theta_k}^*(0) \,|\, \theta_k=\theta_i) \approx \frac{p_{\theta_i}}{t} \sum_{k=1}^{t} \mathbb{P}(\deg_k^+(t) < L_{\theta_k}^*(0) \,|\, \theta_k=\theta_i) \\
&\overset{(e)}{\approx} \frac{p_{\theta_i}}{t} \frac{L_{\theta_i}^*(0)}{p_{\theta_i}} \overset{(f)}{=} \frac{L_{\theta_i}^*(0)}{t}.
\end{aligned} \tag{I.3}
$$

APPENDIX J
PROOF OF COROLLARY 6

We start by proving the first statement of the Corollary. From Theorem 4, we know that for an intolerant society with $\gamma_m = \gamma_k = 0$ we have that

$$
\begin{aligned}
\mathbb{E}[\deg_i^-(t)] &= L_k^*(0) \log(t/(i-1)), \\
\mathbb{E}[\deg_j^-(t)] &= L_m^*(0) \log(t/(j-1)),
\end{aligned} \tag{J.1}
$$

for agents i and j, with $\theta_i = k$ and $\theta_j = m$. Thus, it follows from (J.1) that if $L_k^*(0) > L_m^*(0)$ and $i < j$, then $\mathbb{E}[\deg_i^-(t)] \geq \mathbb{E}[\deg_j^-(t)], \forall t \geq i$, whereas $L_k^*(0) > L_m^*(0)$ and $i > j$, then $\mathbb{E}[\deg_i^-(t)] \geq \mathbb{E}[\deg_j^-(t)], \forall t \geq T^*$, where

$$
T^* = \frac{(i-1)^{\frac{L_k^*(0)}{L_k^*(0)-L_m^*(0)}}}{(j-1)^{\frac{L_m^*(0)}{L_k^*(0)-L_m^*(0)}}}.
$$

The proof for the case of $\gamma_m = \gamma_k = 1$ can be carried out in the exact same manner. The second statement of the Corollary follows straightforwardly from the fact that popularity grows logarithmically in non-opportunistic societies, whereas it grows at least sublinearly in time for opportunistic societies (Theorem 4), thus there always exists a finite time after which the expected popularity of an agent in an opportunistic society exceeds that of an agent in a non-opportunistic society.

APPENDIX K
PROOF OF THEOREM 5

Following [24], [29], we adopt a mean-field approximation for the popularity growth process. That is, we assume that an agent's indegree is deterministic and is given by the expected indegree of that agent (numerical validation and justification of such an assumption can be found in [24], [29]). In this case, the cdf of the popularity of type-k agents at time t, denoted by $F_d^{t,k}(d)$, can be computed as follows (see Lemma 1 in [24])

$$
\begin{aligned}
F_d^{t,k}(d) &= 1 - \frac{|\{i \leq t \,|\, \mathbb{E}[\deg_i^-(t)] \geq d, \theta_i = k\}|}{|\mathcal{V}_k^t|} \\
&= 1 - \frac{i^*(d)}{t},
\end{aligned} \tag{K.1}
$$

where $i^*(d)$ is the solution to the equation $\mathbb{E}[\deg_i^-(t)] = d$. In the case when $\gamma_m = \gamma_k = 0$, we have that (Theorem 4)

$$
\mathbb{E}[\deg_i^-(t)] = L_{\theta_i}^*(0) \log(t/(i-1)),
$$

and hence $i^*(d) = t\, e^{\frac{-d}{L_{\theta_i}^*(0)}}$, and we have that

$$
F_d^{t,\theta}(d) = 1 - e^{\frac{-d}{L_\theta^*(0)}}.
$$

Therefore, for $L_k^*(0) > L_m^*(0)$, we have that $F_d^{t,m}(d) \geq F_d^{t,k}(d), \forall d$, which concludes the first statement of the Theorem. The second statement of the Theorem can be proved in the exact same manner.

APPENDIX L
PROOF OF THEOREM 6

We want to prove that

$$
(\exists k \in \Theta, h_k < 1, \gamma_k < 1) \iff (\mathbb{P}(\lim_{t\to\infty} \omega(G^t) = 1) = 1), \tag{L.1}
$$

which can be broken down to the following sufficiency and necessity statements

$$
(\exists k \in \Theta, h_k < 1, \gamma_k < 1) \Rightarrow (\mathbb{P}(\lim_{t\to\infty} \omega(G^t) = 1) = 1). \tag{L.2}
$$

$$
(\exists k \in \Theta, h_k < 1, \gamma_k < 1) \Leftarrow (\mathbb{P}(\lim_{t\to\infty} \omega(G^t) = 1) = 1). \tag{L.3}
$$

We start by proving the sufficiency condition in (L.2). Assume that the statement $(\exists k \in \Theta, h_k < 1, \gamma_k < 1)$ is true, and assume that at an arbitrary time step τ, the network has 2 disconnected components $\mathcal{C}_1$ and $\mathcal{C}_2$, where $\mathcal{C}_i$ is the set of agents in component i. Let E_{12}^t be the event that a type-k agent who is attached to component $\mathcal{C}_1$ meets a stranger who belongs to component $\mathcal{C}_2$ and links to it, and let E_{21}^t be the event that a type-k agent who is attached to component $\mathcal{C}_2$ meets a stranger who belongs to type $\mathcal{C}_1$ and links to it. Given the definitions of E_{12}^t and E_{21}^t, the following statements hold

$$
\begin{aligned}
(m_i(t) = j \,|\, i \in \mathcal{C}_1, j \in \mathcal{C}_2) &\Rightarrow E_{12}^t, \\
(m_j(t) = i \,|\, i \in \mathcal{C}_1, j \in \mathcal{C}_2) &\Rightarrow E_{21}^t,
\end{aligned} \tag{L.4}
$$

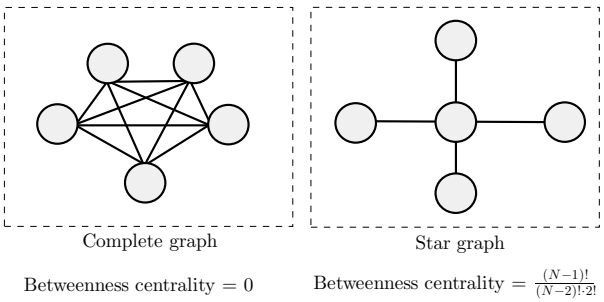


Fig. 13: Network topologies leading to maximum and minimum betweenness centrality.

where $\theta_i = \theta_j = k$. The statements in (L.4) say that agents $i \in \mathcal{C}_1$ and $j \in \mathcal{C}_2$will indeed link if they meet since they are similar-type agents. If follows from (L.4) that

$$\mathbb{P}(E_{12}^t) \geq \mathbb{P}(m_i(t) = j \,|\, i \in \mathcal{C}_1, j \in \mathcal{C}_2),$$
$$\mathbb{P}(E_{21}^t) \geq \mathbb{P}(m_j(t) = i \,|\, i \in \mathcal{C}_1, j \in \mathcal{C}_2). \quad \text{(L.5)}$$

Note that since $i \in \mathcal{C}_1$ and $j \in \mathcal{C}_2$, with $\mathcal{C}_1$ and $\mathcal{C}_2$ being two disconnected network components, then both i and j reside in each others' set of strangers. Hence, we have that for a large network, we have that

$$\mathbb{P}(m_i(t) = j \,|\, i \in \mathcal{C}_1, j \in \mathcal{C}_2) = \frac{p_k(1-\gamma_k)\ |\mathcal{C}_2|}{|\mathcal{C}_1| + |\mathcal{C}_2|},$$
$$\mathbb{P}(m_j(t) = i \,|\, i \in \mathcal{C}_1, j \in \mathcal{C}_2) = \frac{p_k(1-\gamma_k)\ |\mathcal{C}_1|}{|\mathcal{C}_1| + |\mathcal{C}_2|}. \quad \text{(L.6)}$$

That is, the probability that agent i meets j is equal to the probability that agent i meets a stranger (which happens with probability $(1-\gamma_k)$), and that such a stranger belongs to component $\mathcal{C}_2$ (which happens with probability $\frac{|\mathcal{C}_2|}{|\mathcal{C}_1|+|\mathcal{C}_2|}$), and that it is of type-k (which happens with probability p_k). Since $p_k(1-\gamma_k)\frac{|\mathcal{C}_2|}{|\mathcal{C}_1|+|\mathcal{C}_2|} > 0$ and $p_k(1-\gamma_k)\frac{|\mathcal{C}_1|}{|\mathcal{C}_1|+|\mathcal{C}_2|} > 0$, then it follows from (L.5) and (L.6), then it follows that $\mathbb{P}\left(E_{12}^t \vee E_{21}^t\right) > 0, \forall t > \tau$ and hence $\mathbb{P}\left(\bigvee_{t=\tau}^{\infty}\left(E_{12}^t \vee E_{21}^t\right)\right) = 1$. Therefore, if $\exists k \in \Theta, h_k < 1, \gamma_k < 1$, then any two disconnected components in the network will eventually get connected through a type-k agent, and hence the sufficiency condition in (L.2) follows.

Now we prove the converse. Assume that all realizations of an asymptotically large network are fully connected, then it follows that in all such realizations, there exists links across different types of agents, and hence the condition $h_k = 1, \forall k \in \Theta$ cannot be satisfied, and hence it follows that $\exists k \in \Theta, h_k < 1, \gamma_k < 1$.

APPENDIX M
ILLUSTRATION FOR THE FORMATION OF A DOMINANT COALITION

Before explaining the phenomenon under consideration, we point out to the fact that as shown in Figure 13, the minimum average betweenness centrality for an N-node graph is achieved by a complete graph, $\bar{b} = 0$, whereas the maximum average betweenness centrality is achieved by a star graph as depicted in Figure 13, $\bar{b} = \frac{1}{N} \cdot \binom{N}{2}$.

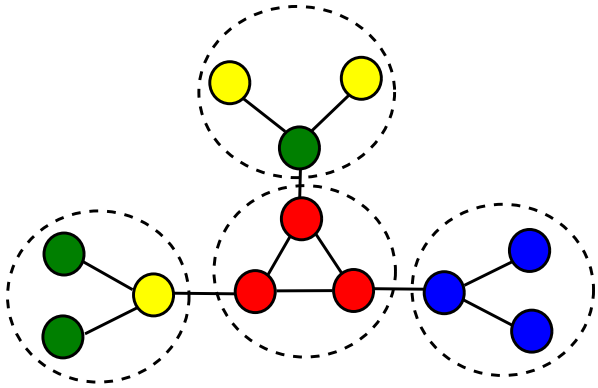


Fig. 14: An exemplary topology for an extremely homophilic group suited in the center of a non-homophilic network.

Now consider a network with four types of nodes: red, blue, green and yellow, with a uniform type distribution and uniform gregariousness among all types (i.e. each agent forms 1 link). The homophily indexes of these types are given by $h_{blue} = h_{yellow} = h_{green} = 0$ and $h_{red} = 1$. That is, the red type is extremely homophilic, whereas other types are extremely non-homophilic. Figure 14 shows an exemplary network structure that can emerge at time $t = 12$. In this exemplary network, agents are equally distributed, the red agents only form links with other red agents, and the other types of agents form links with any other type. We select the exemplary network structure in Figure 14 because: the non-homophilic agents form star sub-graphs (e.g. maximum centrality), and the homophilic agents form a complete sub-graph (e.g. minimum centrality), and such a network structure emerges with a positive probability. Thus, in such a worst case scenario (from the perspective of the homophilic (red) agent), if the homophilic agents are found to be more central than the non-homophilic agents, then it is likely that this would also hold on average for all network paths. This qualitative analysis is not a "proof"; however, the exemplary network realization in Figure 14 is expressive for the source of centrality of the homophilic agents. The average betweenness centrality of all the four types are given by:

$$\bar{b}_{green} = \frac{1}{3} \cdot \binom{3}{2} = 1,$$
$$\bar{b}_{blue} = \frac{1}{3} \cdot \binom{3}{2} = 1,$$
$$\bar{b}_{yellow} = \frac{1}{3} \cdot \binom{3}{2} = 1,$$
$$\bar{b}_{red} > \frac{3 \times 3 + 3 \times 3}{3} = 6,$$

where $\bar{b}_{green}$, $\bar{b}_{blue}$ and $\bar{b}_{yellow}$ are obtained by observing that all non-homophilic agents from star sub-graphs (see Figure 14), whereas $\bar{b}_{red}$ is lower bounded by observing that every two non-homophilic agents in disconnected non-homophilic sub-graphs are bridged by a homophilic agent.

Therefore, $\bar{b}_{red} > \bar{b}_{green}$, $\bar{b}_{red} > \bar{b}_{blue}$, and $\bar{b}_{red} > \bar{b}_{yellow}$, and hence the homophilic agents accrue the maximum bridging capital.

In light of the example given above, we can see that the key insight behind the phenomena of homophilic agents' centrality is that homophilic agents form a central super-node in a star graph of super-nodes as depicted via the dashed circles in Figure 14, and the nodes within this central super-node would naturally exhibit a central position in the network (by analogy with the simple star graph in Figure 13, in which the central node has the maximum betweenness centrality and peripheral nodes have zero centrality). Hence, homophilic agents, suited as a central super-node in a star-like graph, end up bridging agents from different groups, whereas the agents in other groups lie on peripheral positions in the network that exhibits less bridging capital.

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